

Option C Imaging

C1 Introduction to imaging

In this section we discuss the formation of images by lenses and mirrors. We will learn how to construct images graphically as well as algebraically. The section closes with a discussion of two types of lens defect: spherical and chromatic aberration.

C1.1 Lenses

The passage of light rays through lenses is determined by the law of refraction. A ray of light that enters a lens will, in general, deviate from its original path according to **Snell's law** of refraction.

The extent of deviation depends on the index of refraction of the glass making up the lens, the radii of the two spherical surfaces making up the lens, and the angle of incidence of the ray.

We will make the approximation that the lens is always very **thin**, which allows for simplifications. The two sides of the lens need not have the same curvature, and may be convex, concave or planar. Various types of lens are illustrated in Figure C.1.

The straight line that goes through the centre of the lens at right angles to the lens surface is known as the **principal axis** of the lens.

C1.2 Converging lenses

Lenses that are thicker at the centre than at the edges are **converging lenses**, which means that, upon going through the lens, a ray of light changes its direction towards the axis of the lens (see Figure C.2a).

The straight line at right angles to the lens surface and through its centre is called the principal axis of the lens. A beam of rays parallel to the principal axis will, upon refraction through the lens, pass through

Learning objectives

- Work with thin converging and diverging lenses.
- Work with concave and convex mirrors.
- Solve problems with ray diagrams, both graphically and algebraically.
- Understand the difference between real and virtual images.
- Calculate linear and angular magnifications.
- Describe spherical and chromatic aberrations.

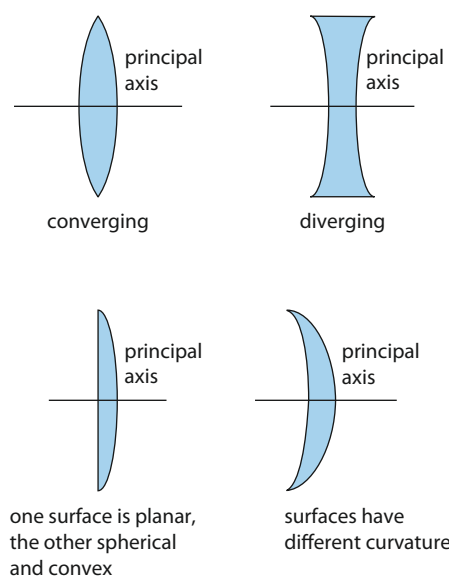


Figure C.1 Various types of lens.

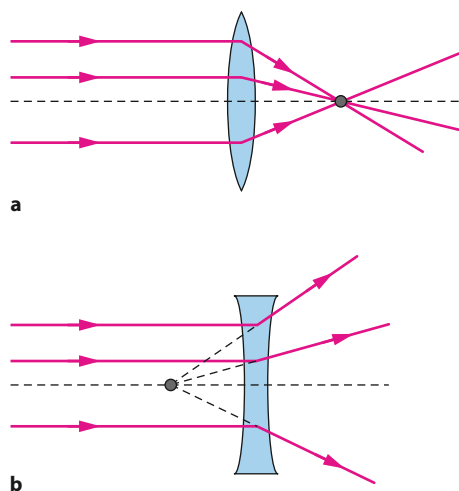


Figure C.2 **a** A converging lens. **b** A diverging lens. A beam of rays parallel to the principal axis converges towards the principal axis in the case of a converging lens but diverges from it in the case of a diverging lens.

the same point on the principal axis on the other side of the lens (see Figure C.3a).

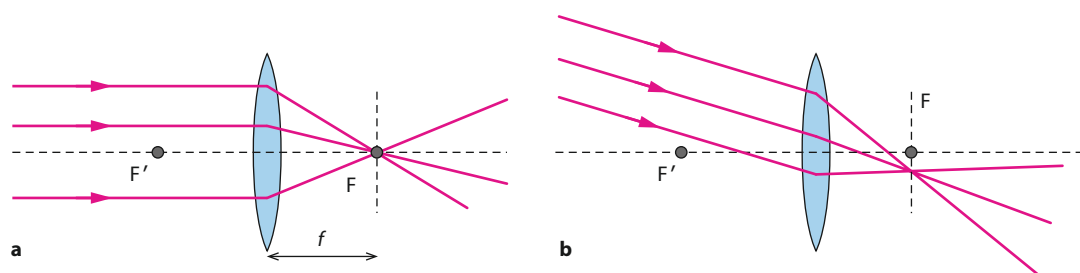


Figure C.3 **a** Upon refraction, rays that are parallel to one another and to the principal axis pass through the focal point of the lens, a point on the principal axis. **b** If the rays are not parallel to the principal axis, they will go through a common point that is in the same plane as the focal point.

Exam tip

Optometrists usually use the inverse of the focal length – called the power, P – to specify a lens:

$$P = \frac{1}{f}$$

If the focal length of a lens is expressed in metres, its power is expressed in dioptres, D ($D = 1 \text{ m}^{-1}$).

For example, a lens with a focal length $f = 25 \text{ cm}$ has a power of $P = \frac{1}{0.25} = 4.0 \text{ D}$

Rays that are parallel to the principal axis will, upon refraction, pass through a point on the principal axis called the **focal point**. The distance of the focal point from the centre of the lens is called the **focal length**, denoted f .

If a parallel beam of rays is not parallel to the axis, the rays will again go through a single point. This point and the focal point of the lens are in the same vertical plane (see Figure C.3b).

(The point on the other side of the lens at a distance f from the lens is also a focal point. A ray parallel to the principal axis and entering the lens from right to left will pass through F' .)

We now know how one set of rays will refract through the lens. Let us call a ray parallel to the principal axis ‘standard ray 1’.

Another ray whose refraction through the lens is easy to describe passes through the left focal point of the lens. It then emerges parallel to the principal axis on the other side of the lens, as shown in Figure C.4. We may call such a ray ‘standard ray 2’.

A third light ray whose behaviour we know something about is directed at the centre of the lens. This ray will go through undeflected, as shown in Figure C.5a. We may call such a ray ‘standard ray 3’.

The reason for this behaviour is that near the midpoint of the lens the two lens surfaces are almost parallel. A ray of light going through glass with two parallel surfaces is shown in Figure C.5b. The ray simply gets shifted parallel to itself. The amount of the parallel shift is proportional to the width of the glass block, which is the thickness of the lens. Since we are making the approximation of a very thin lens, this displacement is negligible.

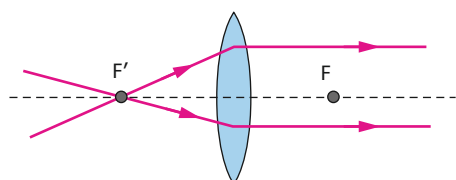


Figure C.4 A ray passing through the focal point emerges parallel to the principal axis.

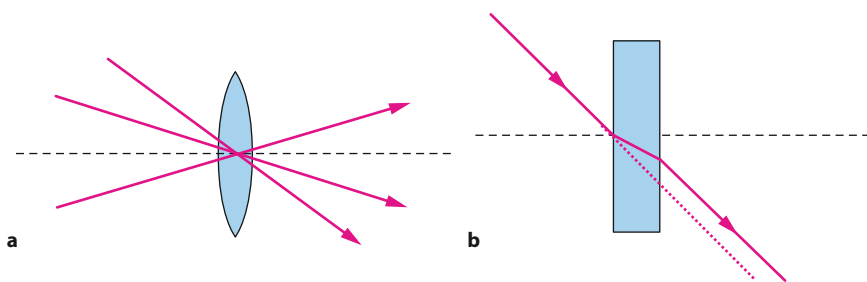


Figure C.5 **a** A ray going through the centre of the lens is undeflected. **b** A ray entering a glass plate is shifted parallel to itself by an amount proportional to the thickness of the plate.

With the help of these three ‘standard rays’ we can find the image of any object placed in front of a converging lens. The standard rays are shown together in Figure C.6.

We will get different kinds of images depending on the distance of the object from the lens. The distances of the object and the image are measured from the centre of the lens.

The convention shown in Figure C.7 for the representation of a lens by a single line, rather than by its actual shape, is helpful in simplifying graphical solutions.

We start with Figure C.8, showing an object 12 cm from the lens. The object is 4.0 cm high. The focal length of the lens is 4.0 cm. We draw the three standard rays leaving the top of the object. These meet at point P on the other side of the lens, and that point is the image of the top of the object. Because the object is at right angles to the principal axis, the rest of the image is found by just drawing a vertical arrow from the principal axis to the point P. We observe that the distance of the image is about 6.0 cm and its height is about 2.0 cm. The image is inverted (upside down).

In our second example, the object is placed in between the lens and the focal point of the lens (Figure C.9).

Here the object distance is 4.0 cm and the focal length is 6.0 cm. We draw the standard rays. Standard ray 2 is awkward: how can we draw this ray passing through the focal point and then refracting through the lens? We do so by imagining a backwards extension that starts at the focal point.

The diagram shows that the three refracted rays do not cross on the other side of the lens. In fact they diverge, moving away from each other. But if we extend these refracted rays backwards, we see that their extensions meet, at point P. An observer on the right side of the lens, seeing the refracted rays, would think that they originated at P. The rest of the image is constructed by drawing a vertical arrow to P from the principal axis. The image is thus formed on the same side of the lens as the object, upright and larger. Its height is 9.0 cm and its distance is 12 cm.

There is an essential difference between the images in Figures C.8 and C.9. In Figure C.8, actual rays pass through the image. In Figure C.9 no rays originate from the image; only the mathematical extensions of the rays do. In the first case a screen placed in the image plane would show the image on the screen. A screen placed in the image plane in the second case would show nothing. The rays of light would increase the temperature at the position of the first image, but no such rise in temperature would occur in the second case.

The first image is called **real** and the second **virtual**.

A **real image** is formed by actual rays and can be projected on a screen. A **virtual image** is formed by extensions of rays and cannot be projected on a screen.

It is left as an exercise to draw the **ray diagram** for an object that is placed at a distance from the lens which is exactly equal to the focal length. You will find that the refracted rays are parallel. In this case neither the rays themselves nor their extensions meet. The image is said to form at infinity.

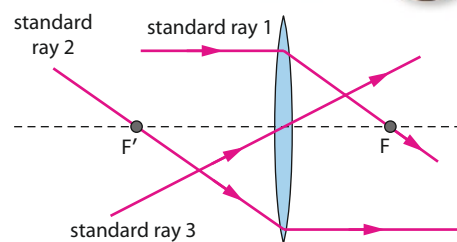


Figure C.6 Refraction of the three standard rays in a converging lens.

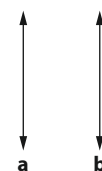


Figure C.7 Convention for representing **a** converging and **b** diverging lenses.

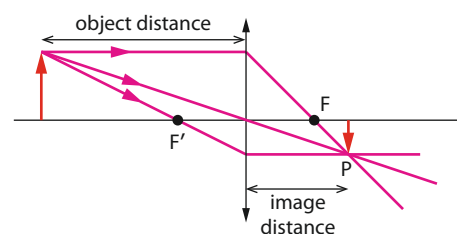


Figure C.8 Image formation in a converging lens. The image is real.

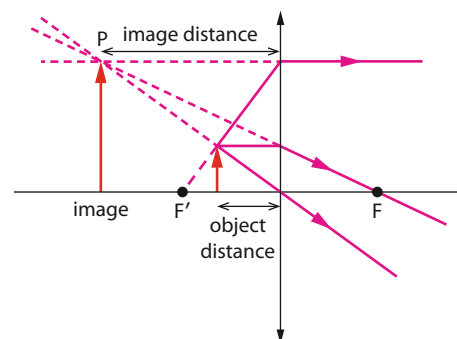


Figure C.9 Image formation in a converging lens. The image is virtual.

Exam tip

You can form the image using just two of the three standard rays, but the third ray provides a check on your drawing.

C.1 Figure **C.10** shows the image of an object in a lens. A ray of light leaves the top of the object. **a** On a copy of this diagram, draw a line to show how this ray refracts in the lens. **b** Draw another line to locate the focal point of the lens.



Figure C.10

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- A ray diagram for a convex lens. An object, represented by an upward arrow, is placed to the left of the lens. Three rays are shown originating from the top of the object: one parallel to the principal axis that refracts through the far focal point, one through the optical center that continues straight, and one through the near focal point that refracts parallel to the axis. These rays diverge on the right side, and their backward extensions intersect to form a larger, upright, virtual image on the same side as the object. The image is labeled 'image' and represented by a downward arrow. The focal point on the right is labeled 'F'.

The methods described above are graphical methods for finding an image. These are very useful because they allow us to ‘see the image being formed’. There is, however, also an algebraic method, which is faster. This uses an equation relating the object and image distances to the focal length of the lens.

A ray diagram for a convex lens forming a real, inverted image. The lens is represented by a blue vertical oval with a dashed vertical line through its center M . The principal axis is a horizontal line passing through M . The object, labeled AB , is an upward arrow at point A on the left side of the lens, with height h . The image, labeled $A'B'$, is a downward arrow at point A' on the right side of the lens, with height h' . The object distance is labeled u as the distance from A to the lens. The focal point F is marked on the right side of the lens. Two rays from the top of the object B are shown: one ray is parallel to the principal axis and refracts through F ; the other ray passes through the optical center M and continues straight. The intersection of these rays forms the top of the image B' . Points P and P' are marked on the upper half of the lens.

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Let u be the distance of the object from the lens and v the distance of the image from the lens. Triangles ABM and A'B'M are similar (their angles are clearly equal), so

$$\frac{h}{u} = \frac{h'}{v} \Rightarrow \frac{h'}{h} = \frac{v}{u}$$

The lens is thin, so P and P' may be considered to be the same point. Then triangles MPF and A'B'F are similar, so

$$\frac{h}{f} = \frac{h'}{v-f} \Rightarrow \frac{h'}{h} = \frac{v-f}{f}$$

(note that MP = h). Combining the two equations gives

$$\frac{v}{u} = \frac{v-f}{f}$$

$$\Rightarrow vf = uv - uf$$

$$\Rightarrow vf + uf = uv \text{ (divide by } uvf \text{)}$$

$$\Rightarrow \frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

The last equation is known as the **thin-lens equation**, and may be used to obtain image distances.

To examine whether the image is larger or smaller than the object, we define the **linear magnification**, m , of the lens as the ratio of the image height to the object height:

$$m = \frac{\text{image height}}{\text{object height}}$$

Numerically, the linear magnification is $m = \frac{v}{u}$. It turns out to be convenient to introduce a minus sign, so we define linear magnification as

$$m = \frac{\text{image height}}{\text{object height}} = \frac{h'}{h} = -\frac{v}{u}$$

The usefulness of the minus sign will be appreciated in the following Worked examples.

The thin-lens equation and the magnification formula allow a complete determination of the image without a ray diagram. But to do this, a number of conventions must be followed:

- f is positive for a converging lens
- u is positive
- v is positive for real images (those formed on the other side of the lens from the object)
- v is negative for virtual images (those formed on the same side of the lens as the object)
- $m > 0$ means the image is upright
- $m < 0$ means the image is inverted
- $|m| > 1$ means the image is larger than the object
- $|m| < 1$ means the image is smaller than the object.

Worked examples

C.2 A converging lens has a focal length of 15 cm. An object is placed 60 cm from the lens. Determine the size of the image and the value of the magnification.

The object distance is $u = 60$ cm and the focal length is $f = 15$ cm. Thus,

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{15} - \frac{1}{60} = \frac{1}{20} \Rightarrow v = 20 \text{ cm}$$

The image is real (positive v) and is formed on the other side of the lens. The magnification is $m = \frac{20}{60} = \frac{1}{3}$. The negative sign in the magnification tells us that the image is inverted. The magnitude of the magnification is less than 1. The image is three times shorter than the object. (Construct a ray diagram for this example.)

C.3 An object is placed 15 cm in front of a converging lens of focal length 20 cm. Determine the size of the image and the value of the magnification.

Applying the lens equation, we have

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{20} - \frac{1}{15} = -\frac{1}{60} \Rightarrow v = -60 \text{ cm}$$

The image is virtual (negative v) and is formed on the same side of the lens as the object. The magnification is $m = -\left(-\frac{60}{15}\right) = +3$. Thus the image is three times taller than the object and upright (positive m).

The lens here is acting as a magnifying glass. (Construct a ray diagram for this example.)

A converging lens can produce a real or a virtual image, depending on the distance of the object relative to the focal length.

C1.3 Diverging lenses

Lenses that are thinner at the centre than at the edges are **diverging lenses**, which means that a ray of light changes its direction away from the axis of the lens (see Figure C.2b). Rays of a parallel beam of light diverge from each other after going through the lens (Figure C.13).

With small but important changes, much of the discussion for converging lenses can be repeated for diverging lenses. We need to know the behaviour of three standard rays in order to construct an image graphically.

First we need a definition of the focal point of a diverging lens. In a diverging lens, rays coming in parallel to the principal axis will, upon refraction, move away from the axis in such a way that their extensions go through a point on the principal axis called the **focal point** of the lens. The distance of the focal point from the centre of the lens is the **focal length** of the diverging lens (see Figure C.13a). This is our 'standard ray 1' for diverging lenses.

If the beam is not parallel to the principal axis, the extensions of the refracted rays will all go through the same point at a distance from the lens equal to the focal length (see Figure C.13b).

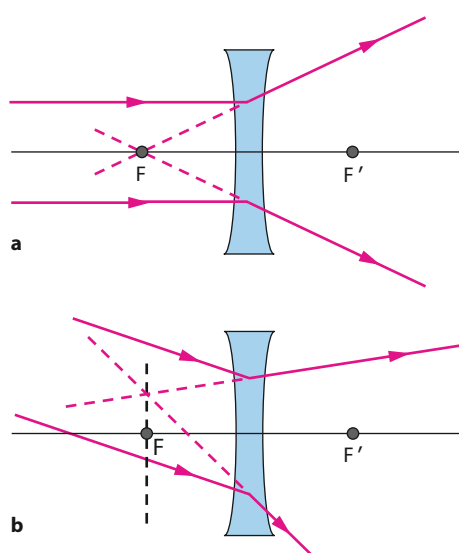


Figure C.13 **a** Rays parallel to the principal axis diverge from the lens in such a way that the extensions of the rays pass through the focal point of the lens on the same side as the incoming rays. **b** Other parallel rays at an angle to the principal axis will appear to come from a point off the principal axis at a distance from the centre of the lens equal to the focal length.

A ray directed at the focal point on the other side of the lens will refract parallel to the principal axis, as shown in Figure C.14. This is 'standard ray 2'.

Finally, a ray that passes through the centre of the lens is undeflected, as shown in Figure C.15. This is 'standard ray 3'.

The behaviour of all three standard rays is shown in Figure C.16.

With this knowledge we can obtain the images of objects placed in front of a diverging lens. Consider an object 8.0 cm in front of a diverging lens of focal length 6.0 cm. The height of the object is 2.0 cm. Using all three standard rays (even though only two are required), we see that an image is formed at about 3.4 cm from the lens. The image is virtual (formed by extensions of rays) and upright, and has a height of about 0.86 cm (see Figure C.17).

Exam tip

In using the lens formula for a diverging lens, the focal length is taken to be negative.

It can be shown that the formula relating object and image distances and focal length that we used for converging lenses applies to diverging lenses as well, with the very important difference that the focal length is taken as negative. The remaining conventions are the same as for converging lenses.

As an example, consider a diverging lens of focal length 10 cm and an object placed 15 cm from the lens. Then

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = -\frac{1}{10} - \frac{1}{15} - \frac{1}{6.0} \Rightarrow v = -6.0 \text{ cm}$$

The negative sign for v implies that the image is virtual and is formed on the same side of the lens as the object. The magnification is

$m = -\left(\frac{-6.0}{10}\right) = +0.60$, implying an upright image 60% of the height of the object.

When the object is real (i.e. $u > 0$), a diverging lens always produces a virtual image ($v < 0$). The magnification is then always positive, implying an upright image.

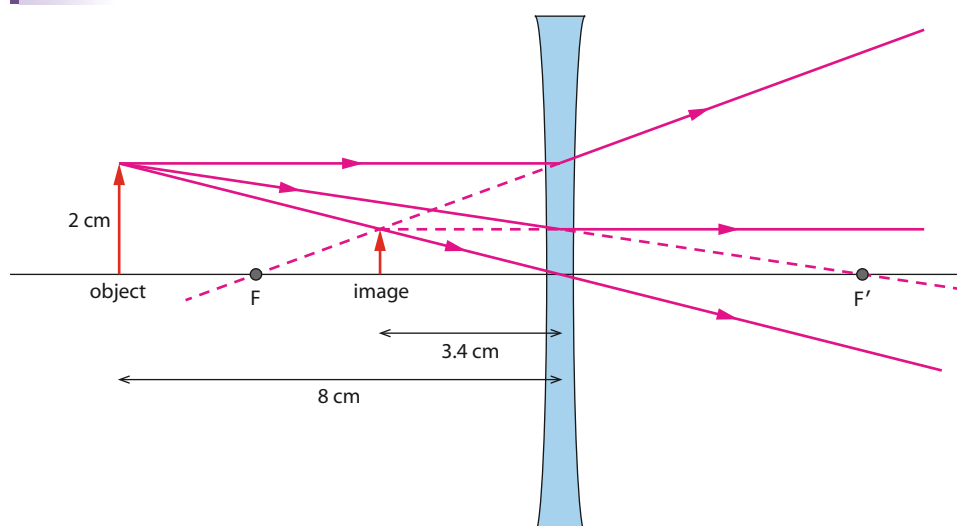


Figure C.17 Formation of an image by a diverging lens. All three standard rays are shown here.

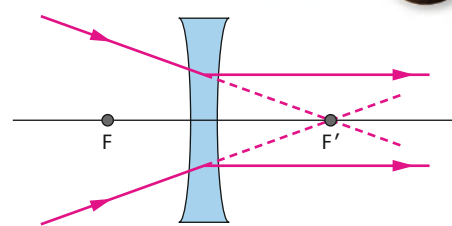


Figure C.14 A ray directed towards the focal point on the other side of the lens emerges parallel to the principal axis.

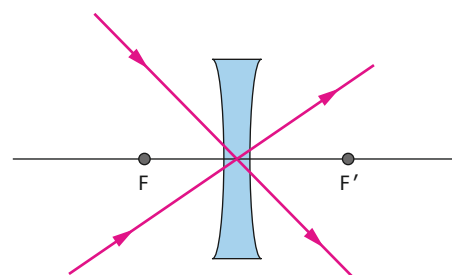


Figure C.15 A ray directed at the centre of the lens passes through undeflected.

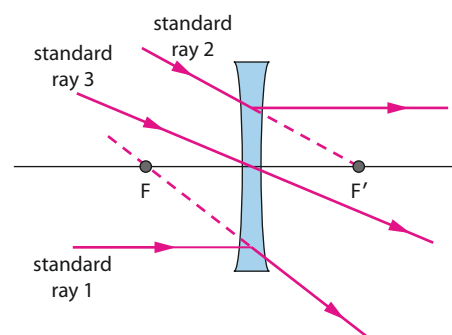


Figure C.16 Refraction of the three standard rays in a diverging lens.

C1.4 Lens combinations: virtual objects

Figure C.18 shows two converging lenses 12 cm apart. The left lens has a focal length of 4.0 cm and the right lens a focal length of 2.0 cm. An object 4.0 cm tall is placed 12.0 cm to the left of the left lens. What are the characteristics of the final image? The answer may be obtained by a ray diagram or algebraically. We begin with the ray diagram.

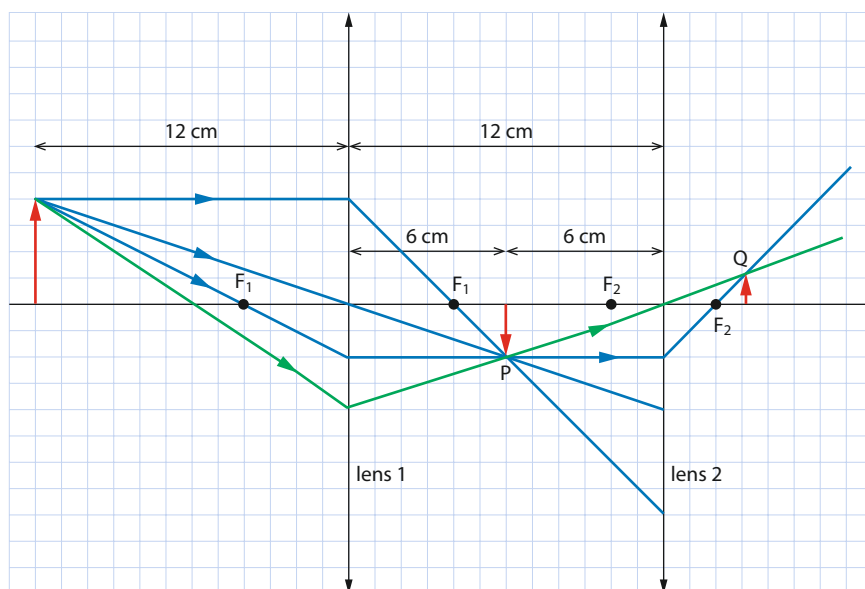


Figure C.18 Formation of a real image by a two-lens system.

We have drawn the three standard rays leaving the object (in blue). Upon refraction through the first lens the rays meet at point P and continue until they reach the second lens. The position of the image in the first lens is at P.

This image now serves as the object for the second lens. The diagram shows the three standard rays of the first lens arriving at the second lens. Of these, only one is also a standard ray for the second lens: the one parallel to the principal axis. We know that this ray, upon refraction in the second lens, will pass through the focal point of the second lens, as shown in Figure C.18. We need another ray through the second lens. We choose the one from P which passes through the centre of the second lens (the green ray), emerges undeflected and meets the blue ray at Q. This is the position of the final image. We see that this image is real, 3.0 cm to the right of the second lens, upright and with a height of 1.0 cm. (In the diagram we have extended the green line backwards to the top of the object.)

These results can also be obtained using the formula. We first find the image in the first lens: we have that $u_1 = 12$ cm and $f_1 = 4.0$ cm, so

$$\frac{1}{v_1} = \frac{1}{f_1} - \frac{1}{u_1} = \frac{1}{4.0} - \frac{1}{12} = \frac{1}{6.0} \Rightarrow v_1 = 6.0 \text{ cm}$$

The magnification of the first lens is

$$m_1 = -\frac{v_1}{u_1} = -\frac{6.0}{12} = -0.50$$

This means that the image is half the size of the object and inverted. This is what the ray diagram shows as well. This image now serves

as the object for the second lens. The distance of this object from the second lens is $12 - 6.0 = 6.0$ cm. Hence the new (and final) image is at

$$\frac{1}{v_2} = \frac{1}{f_2} - \frac{1}{u_2} = \frac{1}{2.0} - \frac{1}{6.0} = \frac{1}{3.0} \Rightarrow v_2 = 3.0 \text{ cm}$$

The magnification of the second lens is

$$m_2 = -\frac{v_2}{u_2} = -\frac{3.0}{6.0} = -0.50$$

This means that the image is inverted relative to its object. But the object is already inverted, so the final image is upright. It is 50% as high as the object, or 1.0 cm tall. Overall, the image is four times smaller than the original object. This is because the overall magnification of the two-lens system is $m = m_1 m_2 = -0.50 \times (-0.50) = +0.25$.

We now consider a slightly more involved example. We again have two converging lenses 8.0 cm apart. The focal lengths are 6.0 cm and 4.0 cm for the left and right lens, respectively. The object is again 12 cm from the left lens (Figure C.19).

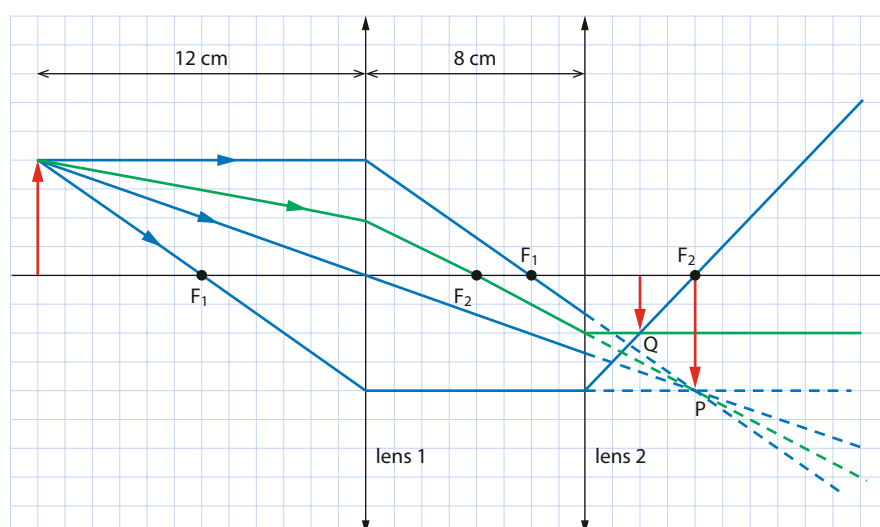


Figure C.19 Formation of a real image by a two-lens system.

We again draw the three standard rays (blue) leaving the top of the object. If the second lens were not there, these rays would meet at P and the image in the first lens would form there. Instead, the three blue rays arrive at the second lens. Of these, only one is a standard ray for the second lens: the one that is parallel to the principal axis, which will pass through the focal point to the right of the second lens. We need one more ray from the top of the original object, and choose the green ray, through focal point F_2 and point P. Since this ray passes through F_2 it will refract parallel to the principal axis. This ray intersects the blue line at Q, and this is the position of the final image. We see that the image is 2.0 cm to the right of the right lens, upright and 2.0 cm tall, and is a real image.

How do we get the same results with the formula? Applying it to the first lens, we have that $u_1 = 12$ cm and $f_1 = 6.0$ cm, so

$$\frac{1}{v_1} = \frac{1}{f_1} - \frac{1}{u_1} = \frac{1}{6.0} - \frac{1}{12} = \frac{1}{12} \Rightarrow v_1 = 12 \text{ cm}$$

Exam tip

Let h_1 be the height of the original object, h_2 the height of the image in the first lens and h_3 the height of the final image. The overall magnification is

$$m = \frac{h_3}{h_1} = \frac{h_3}{h_2} \times \frac{h_2}{h_1} = m_1 m_2,$$

which is the product of the magnifications for the individual lenses.

The image is 12 cm to the right of the first lens (or 4.0 cm to the right of the second lens). The magnification is

$$m_1 = -\frac{v_1}{u_1} = -\frac{12}{12} = -1.0$$

This means that the image is the same size as the object and inverted, consistent with our ray diagram.

Now this image serves as the object for the second lens. However, it is on the 'wrong' side of the lens! This makes this image a 'virtual object' for the second lens, and we must take its sign as negative in the formula for the second lens: $u_2 = -4.0$ cm.

$$\frac{1}{v_2} = \frac{1}{f_2} - \frac{1}{u_2} = \frac{1}{4.0} - \left(-\frac{1}{12}\right) = \frac{1}{2.0} \Rightarrow v_2 = 2.0 \text{ cm}$$

The magnification of the second lens is

$$m_2 = -\frac{v_2}{u_2} = -\frac{2.0}{-4.0} = +0.50$$

This means that the image is not inverted relative to its object. But that object is already inverted, so the final image is inverted, relative to the original object. It is half as tall as the object, or 2.0 cm tall, and thus half as tall as the original object. This is because the overall magnification of the two-lens system is $m = m_1 m_2 = -1.0 \times 0.50 = -0.50$.

Worked examples

C.4 An object lies on a table. A converging lens of focal length 6.0 cm is placed 4.0 cm above the object.

- Determine the image formed by this lens.
- A second converging lens of focal length 5.0 cm is now placed 3.0 cm above the first lens. Determine the image formed by this combination of lenses.

a With just the first lens, the image is formed at a distance found from

$$\begin{aligned} \frac{1}{v_1} + \frac{1}{u_1} &= \frac{1}{f_1} \\ \frac{1}{v_1} &= \frac{1}{6.0} - \frac{1}{4.0} = -\frac{1}{12} \\ v_1 &= -12 \text{ cm} \end{aligned}$$

$$\text{The magnification is } m_1 = -\frac{v_1}{u_1} = -\frac{-12}{4.0} = +3.0$$

The image is virtual, upright and three times taller.

b This image acts as the object for the second lens. Its distance from the second lens is $12 + 3.0 = 15$ cm. It is a real object for the second lens, so $u_2 = +15$ cm. The new image is thus formed at a distance found from

$$\begin{aligned} \frac{1}{15} + \frac{1}{v_2} &= \frac{1}{5.0} \\ v_2 &= 7.5 \text{ cm} \end{aligned}$$

$$\text{The final image is thus real. The magnification of the second lens is } m_2 = -\frac{v_2}{u_2} = -\frac{7.5}{15} = -0.50$$

The overall magnification is $m = m_1 m_2 = 3.0 \times (-0.50) = -1.5$. Thus the final image is inverted and 1.5 times as tall.

C.5 An object is placed 8.0 cm to the left of a converging lens of focal length 4.0 cm. A second diverging lens of focal length 6.0 cm is placed 4.0 cm to the right of the converging lens. Determine the image of the object in the two-lens system, and verify your results with a scaled ray diagram.

The image in the converging lens is found from

$$\frac{1}{8.0} + \frac{1}{v_1} = \frac{1}{4.0} \Rightarrow v_1 = 8.0 \text{ cm}$$

Its distance from the diverging lens is therefore 4.0 cm. This image acts as a virtual object for the diverging lens. Hence $u_2 = -4.0$ cm. The final image is therefore at a distance found from

$$\frac{1}{-4.0} + \frac{1}{v_2} = \frac{1}{-6.0} \Rightarrow v_2 = 12 \text{ cm}$$

The image is thus real. The magnification of the lens system is $\left(\frac{-8.0}{8.0}\right) \times \left(\frac{-12}{-4.0}\right) = -3.0$, which implies that the final image is inverted and three times as large as the original object. Figure C.20 is a ray diagram of the problem.

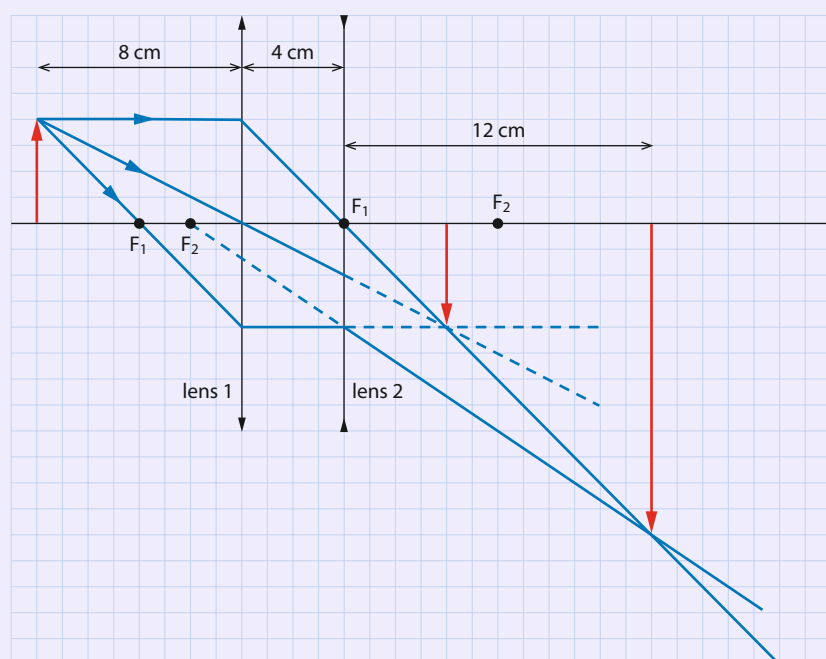


Figure C.20

C1.5 Wavefronts and lenses

Since a lens changes the direction of rays refracting through it, wavefronts (which are normal to rays) also change shape. Figure C.21 shows plane wavefronts in air approaching and entering a transparent surface that has a curved boundary.

The wavefront AC first reaches the curved boundary at point A. In one period, point A will move forward a distance equal to one wavelength in the new medium, where the speed of light is less than in air. The wavelength in the new medium will be shorter than in air. Point A will therefore move to point B. On the other hand, point C will move a longer distance in air and get to point D. Points B and D are part of the new wavefront. We see that it has to curve. The way the wavefronts curve implies that rays converge towards point F in the new medium and, from there on, the rays diverge.

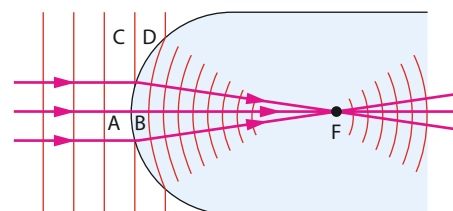


Figure C.21 Plane wavefronts curve after entering another medium with a curved boundary.

In similar fashion, we can see how wavefronts curve as they pass through converging and diverging lenses (Figure C.22).

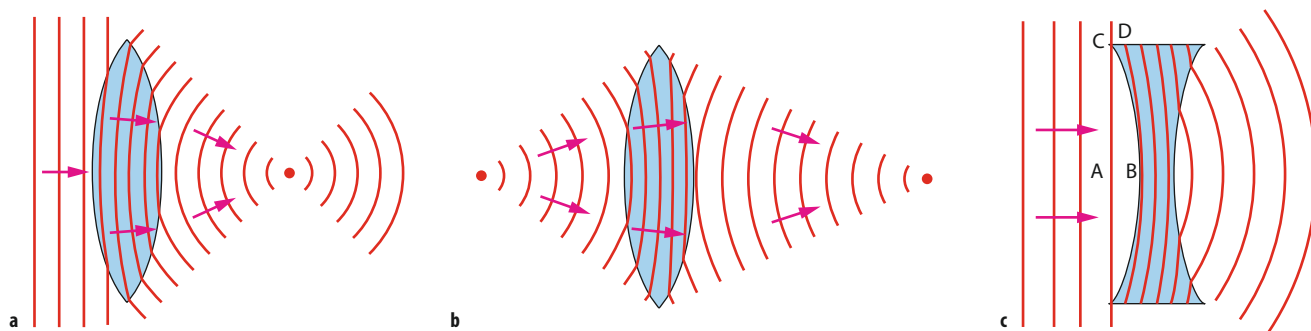


Figure C.22 **a** Plane wavefronts moving through a converging lens. **b** Spherical wavefronts moving through a converging lens. **c** Plane wavefronts moving through a diverging lens.

C1.6 Mirrors

Much of what we have learned about lenses also applies to mirrors. The big difference, of course, is that here the phenomenon is reflection of rays off a mirror surface, not refraction through a lens. We will first deal with spherical mirrors, whose surfaces are cut from a sphere.

We distinguish between concave and convex mirrors. With concave mirrors (Figure C.23a), rays parallel to the principal axis reflect through a common point on the principal axis – the focus of the mirror. With convex mirrors (Figure C.23b), rays parallel to the principal axis reflect such that their extensions go through a common point on the principal axis, behind the mirror – the focus of the convex mirror.

Figure C.24 shows how the standard rays reflect off concave and convex mirrors. Ray 1 is parallel to the principal axis. It reflects such that the ray or its extension goes through the focal point. Ray 2 goes through the focal point (or its extension does), and reflects parallel to the principal axis. Ray 3 is directed at the centre of the mirror and reflects so as to make the same angle with the principal axis as the incident ray.

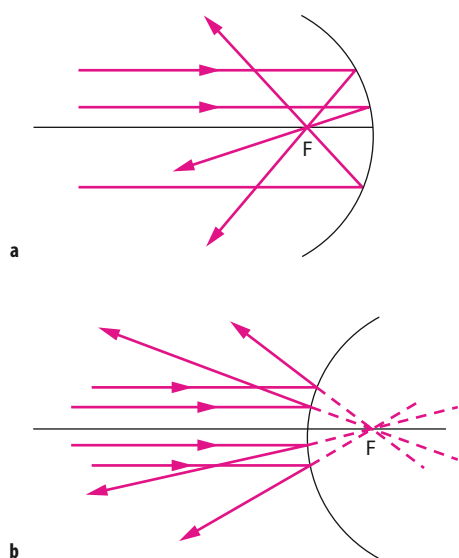


Figure C.23 **a** Concave and **b** convex spherical mirrors.

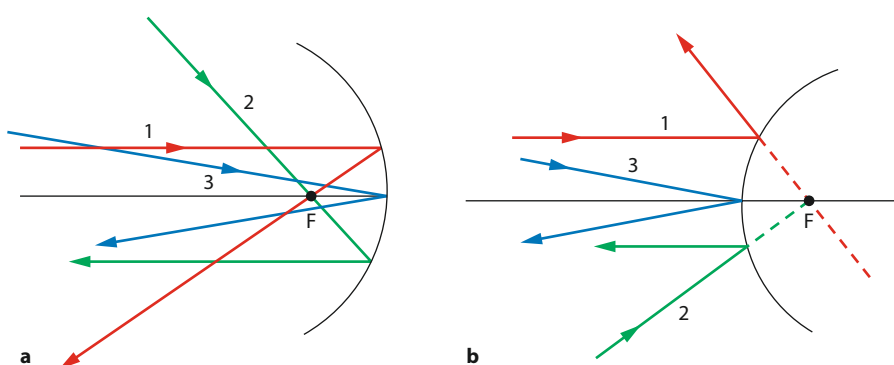


Figure C.24 The standard rays in **a** concave and **b** convex spherical mirrors.

The formula relating object and image distances to focal length that we learned for lenses also applies to mirrors, along with the same conventions. The formula for magnification is also the same. For convex mirrors (as with diverging lenses), the focal length is taken to be negative.

Worked example

C.6 Make sure you understand the formation of these images in the concave mirror in Figure C.25. Explain which case creates a real image and which a virtual image.

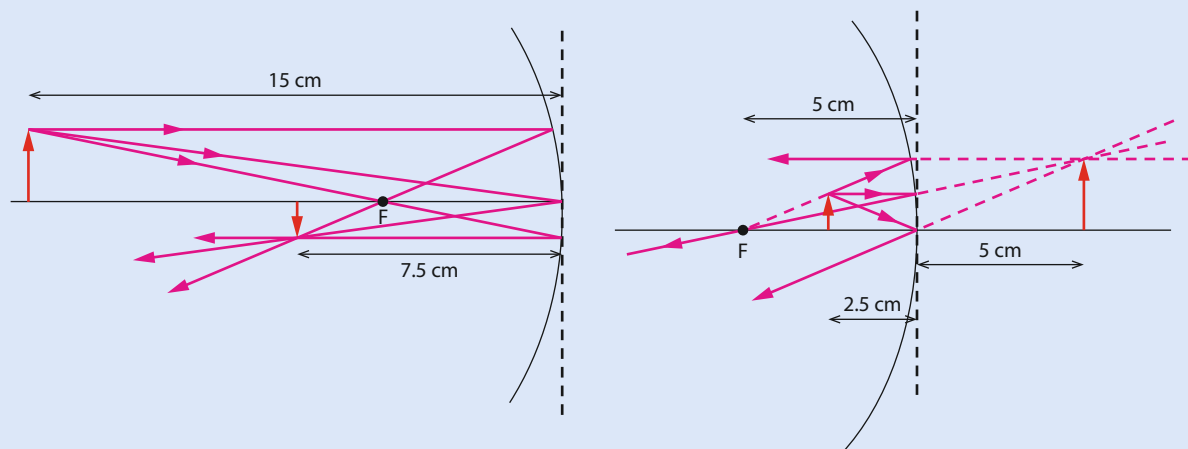


Figure C.25

The image is real in the left diagram because it is formed by real rays. The image is virtual in the second case because it is formed by ray extensions.

The statement that rays parallel to the principal axis of a spherical mirror reflect through the same point on the principal axis (the focal point) is strictly true only for rays that are very close to the principal axis; these are called **paraxial** rays. All rays parallel to the principal axis can be made to reflect through the same point using mirrors with a parabolic shape (Figure C.26).

For such mirrors, rays parallel to the axis reflect through the focal point no matter how far they are from the axis. Rays from the Sun arrive parallel to one another, and a parabolic mirror can focus them and raise the temperature at the focus, enough to heat water or even light the Olympic torch (Figure C.27).

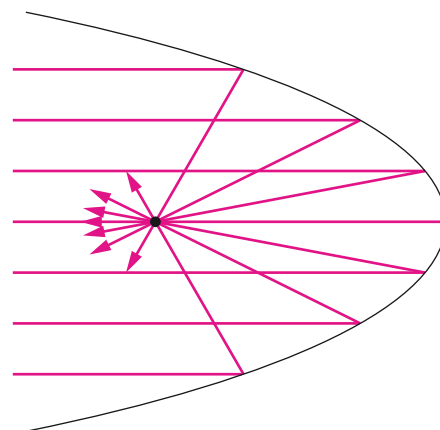


Figure C.26 With a parabolic mirror, all rays parallel to the principal axis reflect through the same point.



Figure C.27 Lighting the Olympic torch at Olympia in Greece, using a parabolic mirror to focus the rays of the Sun.

C1.7 The magnifier

The human eye can produce a clear, sharp image of any object whose distance from the eye is anything from (practically) infinite up to a point called the **near point**. Objects closer to the eye than the near point produce blurry images or force the eye to strain.

The closest point on which the human eye can focus without straining is known as the **near point** of the eye. The distance D of the near point from the eye is about 25 cm for a normal eye, but depends greatly on the age of the person involved.

The closer one gets to an object, the larger the object appears. But of course the object does not change size as you get closer! What makes it appear larger is that the angle the object subtends at the eye gets larger; this creates a larger image on the retina, which the brain interprets as a larger object.

Thus, let an observer view a small object at the near point, $D = 25$ cm from the eye, and let θ be the angle that the object subtends at the eye, as shown in Figure C.28. We know that $\tan \theta = \frac{h}{D}$, but for very small angles $\tan \theta \approx \theta$, so $\theta \approx \frac{h}{D}$.

Let us now view the object through a lens; we place the object very close to the focal point of the lens, in between the focal point and the lens. A virtual, upright, enlarged image will be formed very far from the lens (Figure C.29). The eye can then see the image comfortably and without straining; the eye is relaxed.

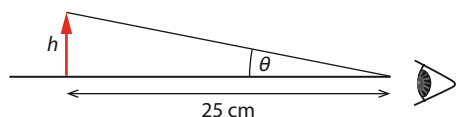


Figure C.28 The apparent size of an object depends on the angle subtended at the eye.

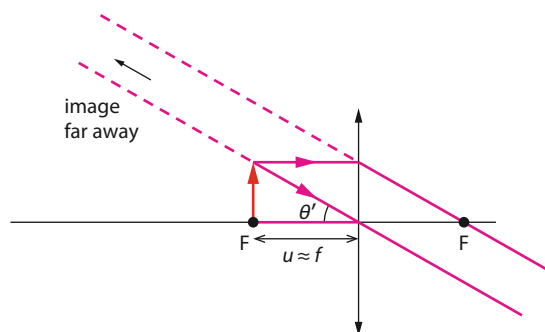


Figure C.29 A converging lens acting as a magnifier. The image is formed very far from the lens when the object is just to the right of the focal point.

Exam tip

Strictly speaking, $\tan \theta = \frac{h}{D}$.

If the angle (in radians) is very small, then $\tan \theta \approx \theta$, so

$\theta \approx \frac{h}{D}$. This is the small-angle approximation.

Exam tip

This is the formula for angular magnification when the image is formed far from the lens (practically at infinity).

We define the **angular magnification** as the ratio of the angle subtended at the eye by the image to the angle subtended by the object when viewed by the unaided eye at the near point (Figure C.28):

$$M = \frac{\theta'}{\theta}$$

Using the small-angle approximation, $\theta = \frac{h}{D}$ and $\theta' = \frac{h}{u}$. But $u \approx f$, so $\theta' \approx \frac{h}{f}$.

$$M \approx \frac{\frac{h}{f}}{\frac{h}{D}} \approx \frac{D}{f}$$

We could, however, arrange to position the object at the right place so that the image is formed at the near point (Figure C.30).

In this case the image is formed at $v = -D$ (the image is virtual, hence the minus sign), so the distance of the object from the lens is given by

$$\frac{1}{u} + \frac{1}{-D} = \frac{1}{f}$$

$$u = \frac{Df}{D+f}$$

Let θ' be the angle that the image subtends at the eye through the lens. From simple geometry we obtain

$$\theta' = \frac{h}{u} = \frac{h(D+f)}{D}$$

and $\theta \approx \frac{h}{D}$ as usual. The angular magnification M is then

$$M = \frac{\theta'}{\theta} = \frac{\frac{h(D+f)}{Df}}{\frac{h}{D}} = \frac{D+f}{f} = 1 + \frac{D}{f}$$

In both cases, the magnification can be increased by decreasing the focal length of the lens. Lens defects known as **aberrations** (see Section C1.8) limit the angular magnification to about 4.

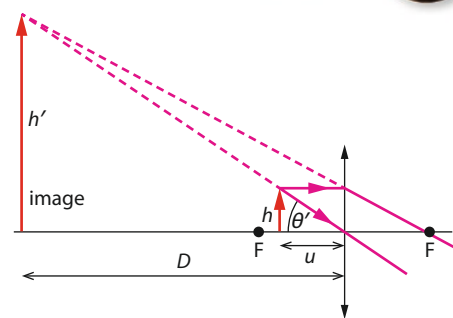


Figure C.30 A converging lens acting as a magnifier. The image is formed at the near point of the eye.

Exam tip

This is the formula for angular magnification when the image is formed at the near point.

Worked example

C.7 An object of length 4.0 mm is placed in front of a converging lens of focal length 6.0 cm. A virtual image is formed 30 cm from the lens.

- Calculate the distance of the object from the lens.
- Calculate the length of the image.
- Calculate the angular magnification of the lens.

a The object distance is found from the lens formula $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$. The image is virtual, so we must remember that $v = -30$ cm. Thus $\frac{1}{u} + \frac{1}{-30} = \frac{1}{6.0}$, so $u = 5.0$ cm.

b The linear magnification is $m = -\frac{v}{u} = -\frac{-30}{5.0} = +6.0$. The length of the object is therefore $6.0 \times 4.0 = 24$ mm.

c The angular magnification is $M = \frac{\theta'}{\theta} = \frac{\frac{h}{u}}{\frac{h}{D}} = \frac{D}{u} = \frac{30}{5.0} = 6.0$. The point of this is to show that you must be careful

with the formulas in the booklet. They apply to the image at infinity or at the near point. For other cases, as here, you have to work from first principles. (We can also find the image height h' from similar triangles. See Figure C.30: $\frac{h'}{30} = \frac{h}{u} \Rightarrow h' = 4.0 \times \frac{30}{5.0} = 24$ mm.)

C1.8 Lens aberrations

Lenses and mirrors do not behave exactly as described above – they suffer from **aberrations**: deviations from the simple description we have provided here. Two main types of aberration are important for lenses: **spherical** and **chromatic**.

Spherical aberration occurs because rays that enter the lens far from the principal axis have a slightly different focal length from rays entering near the axis.

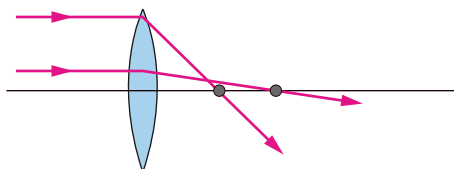


Figure C.31 Spherical aberration: rays far from the axis have a different focal point than those close to the axis, so the image is not a point. (The diagram is exaggerated.)

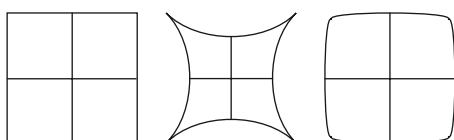


Figure C.32 An example of distortion due to spherical aberration. The grid is distorted because the magnification varies as one moves away from the principal axis.

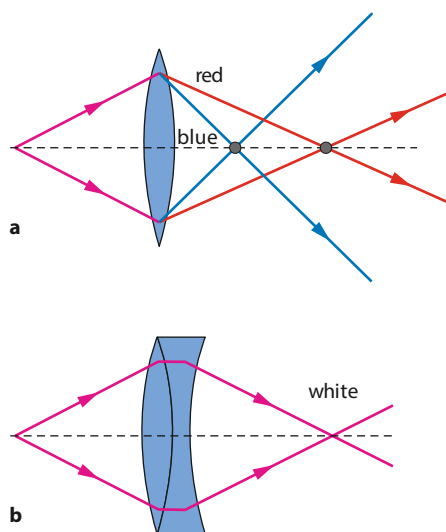


Figure C.33 a Light of different colours bends by different amounts, so different colours are focused at different places.
b An achromatic doublet consists of a pair of lenses of different indices of refraction.

In Figure C.31, rays incident on the lens far from its centre refract through a point on the principal axis that is closer to the lens than rays incident closer to the centre. This means that the image of the point is not a point but a blurred patch of light. Spherical aberration can be reduced by reducing the aperture of the lens (its diameter); this is called **stopping down**. But that means that less light goes through the lens, which results in a less bright image. And a lens with a smaller diameter would also suffer from more pronounced diffraction effects.

The fact that the focal point varies for rays that are further from the principal axis means that the magnification produced by the lens also varies. This leads to a **distortion** of the image, as shown in Figure C.32. Mirrors suffer from spherical aberration just as lenses do.

Chromatic aberration arises because the lens has different refractive indices for different wavelengths. Thus, there is a separate focal length for each wavelength (colour) of light.

This makes images appear faintly coloured – there are lines around the image in the colours of the rainbow (see Figure C.33a).

Of course, chromatic aberration disappears when monochromatic light is used. Chromatic aberration can also be reduced by combining lenses. A diverging lens with a different index of refraction placed near the first lens can eliminate the aberration for two colours and reduce it for the others (see Figure C.33b). Mirrors do not suffer from this type of aberration.

Nature of science

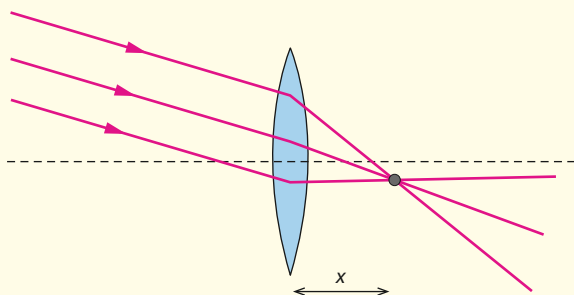
Deductive logic

What does ‘seeing an object’ mean? It means that rays from the object enter the eye, refract in the eye and finally form an image on the retina. Nerves at the back of the retina create electrical signals that are sent to the brain, and the brain reconstructs this information to create the sensation of seeing. But the rays do not necessarily have to come directly from the object and into the eye. The rays may first be reflected off a mirror or refracted through a lens. When the image formed by the mirror or lens is virtual, the brain interprets the rays as originating from a place where no actual, real object exists. Analysis of lenses and mirrors depends on this idea of the virtual image.

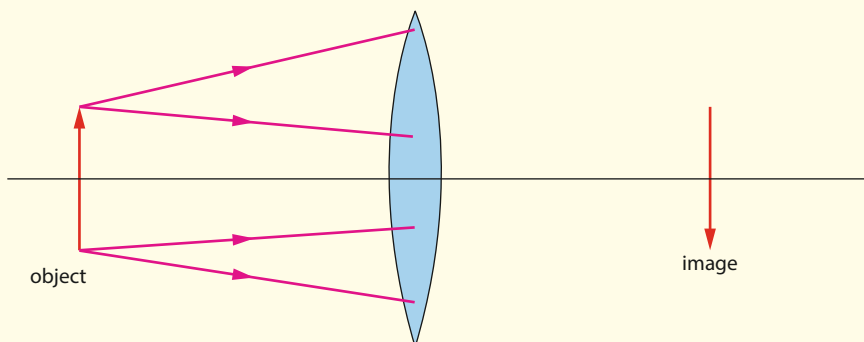


? Test yourself

- Define:
 - the **focal point of a converging lens**
 - the **focal length of a diverging lens**.
- Explain what is meant by:
 - a **real image** formed by a lens
 - a **virtual image** formed by a lens.
- Explain why a real image can be projected on a screen but a virtual image cannot.
- A plane mirror appears to reverse left and right. Does a lens do the same? Explain your answer.
- A converging lens has a focal length of 6.0 cm. Determine the distance x .

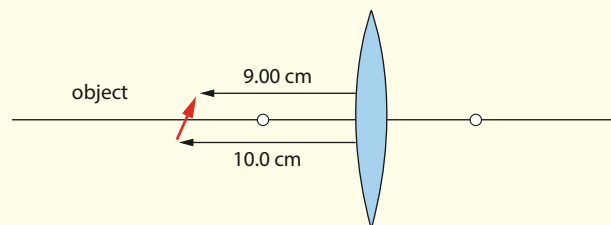


- The diagram below shows the real image of an object in a converging lens. Copy the diagram and complete the rays drawn.



- An object 2.0 cm tall is placed in front of a converging lens of focal length 10 cm. Using ray diagrams, construct the image when the object is at a distance of:
 - 20 cm
 - 10 cm
 - 5.0 cm.
 Confirm your ray diagrams by using the lens equation.
- Using a ray diagram, determine the image characteristics of an object of height 2.5 cm that is placed 8.0 cm in front of a converging lens of focal length 6.0 cm. Confirm your ray diagram by using the lens equation.

- Using a ray diagram, determine the image characteristics of an object of height 4.0 cm that is placed 6.0 cm in front of a converging lens of focal length 8.0 cm. Confirm your ray diagram by using the lens equation.
- A converging lens of focal length 4.5 cm produces a real image that is the same size as the object. Determine the distance of the object from the lens.
- Consider a converging lens of focal length 5.00 cm. An object of length 2.24 cm is placed in front of it, as shown below (not to scale), so that the middle of the object is on the principal axis. By drawing appropriate rays, determine the image in the lens. Is the angle the image makes with the principal axis the same as that for the object?

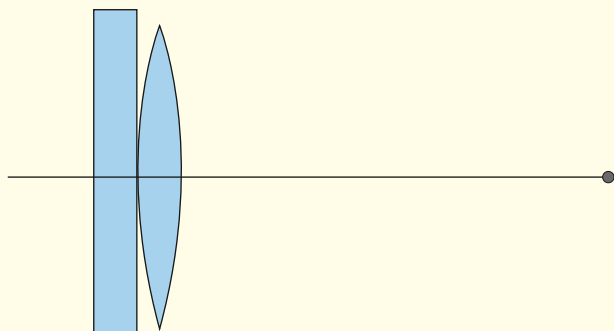


- A student finds the position of the image created by a converging lens for various positions of the object. She constructs a table of object and image distances.

| | | | | | |
|----------------------------------|------|------|------|------|------|
| $u/\text{cm} \pm 0.1 \text{ cm}$ | 12.0 | 16.0 | 20.0 | 24.0 | 28.0 |
| $v/\text{cm} \pm 0.1 \text{ cm}$ | 60.0 | 27.2 | 19.9 | 17.5 | 16.8 |

- Explain how these data can be used to determine the focal length of the lens.
- Determine the focal length, including the uncertainty in its value.

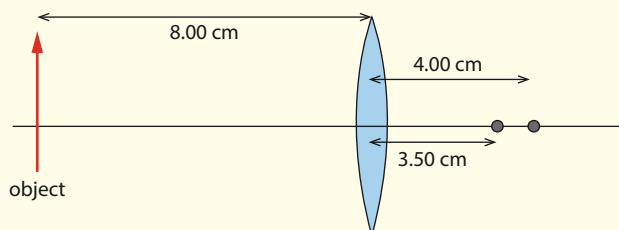
- 13** An object is placed in front of a converging lens which rests on a plane mirror, as shown below. The object is moved until the image is formed exactly at the position of the object itself. Draw rays from the object to form the image in this case. Explain how the focal length of the lens can be determined from this arrangement.



- 14** A converging lens has a focal length of 15 cm. An object is placed 20 cm from the lens.
- Determine the image (i.e. its position and whether it is real or virtual, and upright or inverted) and find the magnification.
 - Draw a ray diagram to confirm your results.
- 15** An object is 5.0 m from a screen. A converging lens of focal length 60 cm is placed between the object and the screen so that an image of the object is formed on the screen.
- Determine the distances from the screen where the lens could be placed for this to happen.
 - Determine which choice results in the larger image.
- 16** An object is placed 12 cm in front of a diverging lens of focal length 4.0 cm. Determine the properties of the image algebraically and with a ray diagram.
- 17** Two very thin lenses of focal lengths f_1 and f_2 are placed in contact. Show that the focal length of the two-lens system is given by $f = \frac{f_1 f_2}{f_1 + f_2}$.
- 18** Two converging lenses, each of focal length 10.0 cm, are 4.00 cm apart. Find the focal length of this lens combination.
- 19** An object is viewed through a system of two converging lenses, L_1 and L_2 (L_2 to the right of L_1). L_1 has a focal length of 15.0 cm and L_2 has a focal length of 2.00 cm. The distance between the lenses is 25.0 cm and the distance between the object (placed to the left of L_1) and L_1 is 40.0 cm. Determine:
- the position of the image
 - the magnification of the image
 - the orientation of the image.
- 20** An object is viewed through a system of two lenses, L_1 and L_2 (L_2 to the right of L_1). L_1 is converging and has a focal length of 35.0 cm; L_2 is diverging and has a focal length of 20.0 cm. The distance between the lenses is 25.0 cm and the distance between the object (placed to the left of L_1) and L_1 is 30.0 cm. Determine:
- the position of the image
 - the magnification of the image
 - the orientation of the image.
- 21**
 - An object is placed 4.0 cm in front of a concave mirror of focal length 12 cm. Determine the properties of the image.
 - Repeat part **a** when the concave mirror is replaced by a convex mirror of the same focal length.
 - In each case draw a ray diagram to show the construction of the image.
- 22** An object that is 15 mm high is placed 12 cm in front of a mirror. An upright image that is 30 mm high is formed by the mirror. Determine the focal length of the mirror and whether the mirror is concave or convex.



- 23 a Describe the two main lens aberrations and indicate how these can be corrected.
- b In an attempt to understand the distortion caused by spherical aberration, a student considers the following model. She places an object of height 4.00 cm a distance of 8.00 cm from a converging lens. One end of the object is 1.00 cm below the principal axis and the other 3.00 cm above. She assumes that rays leaving the bottom of the object will have a focal length of 4.00 cm and the rays from the top a focal length of 3.50 cm (see diagram below).
- i Under these assumptions, draw rays from the bottom and top of the object to locate the image.
- ii Draw the image again by using a 4.00 cm focal length for all rays, and compare.



- 24 An object is placed in front and to the left of a converging lens, and a real image is formed on the other side of the lens. The distance of the object from the left focal point is x and the distance of the image from the right focal point is y . Show that $xy = f^2$.

- 25 A converging lens of focal length 10.0 cm is used as a magnifying glass. An object whose size is 1.6 mm is placed at some distance from the lens so that a virtual image is formed 25 cm in front of the lens.
- a Calculate the distance between the object and the lens.
- b Suggest where the object should be placed for the image to form at infinity.
- c Find the angular size of the image at infinity.
- 26 Angular magnification, for a magnifying glass, is defined as $M = \frac{\theta'}{\theta}$.
- a By drawing suitable diagrams, show the angles that are entered into this formula.
- b A simple magnifying glass produces an image at the near point. Explain what is meant by 'near point'.
- c Show that when a simple magnifying glass produces an image at the near point, the magnification is given by $M = 1 + \frac{25}{f}$, where f is the focal length of the lens in cm.
- 27 The normal human eye can distinguish two objects 0.12 mm apart when they are placed at the near point. A simple magnifying glass of focal length 5.00 cm is used to view images at the near point. Determine how close the objects can be and still be distinguished.

C2 Imaging instrumentation

We owe much of our knowledge about the natural world to optical instruments based on mirrors and lenses. These have enabled the observation of very distant objects through telescopes and very small objects through microscopes. We have already seen how a single converging lens can produce an enlarged upright image of an object placed closer to the lens than the focal length, thus acting as a magnifying glass. The apparent size of an object depends on the size of the image that is formed on the retina. In turn, this size depends on the angle subtended by the object at the eye. This is why we bring a small object closer to the eye in order to view it – the angle subtended at the eye by the object increases.

C2.1 The optical compound microscope

A **compound microscope** (Figure C.34) consists of two converging lenses. It is used to see enlarged images of very small objects. The object (of height h) is placed at a distance from the first lens (the **objective**)

Learning objectives

- Describe and solve problems with compound microscopes.
- Describe and solve problems with astronomical refracting and reflecting telescopes.
- Outline the use of single-dish radio telescopes.
- Understand the principle of radio interferometry telescopes.
- Appreciate the advantages of satellite-borne telescopes.

which is slightly greater than the focal length f_o of the objective. A real inverted image of height h' is formed in front of the second lens (the **eyepiece**) and to the right of the focal point F_e of the eyepiece. This image serves as the object for the eyepiece lens, which, acting as a magnifier, produces an enlarged, virtual image of height h'' .

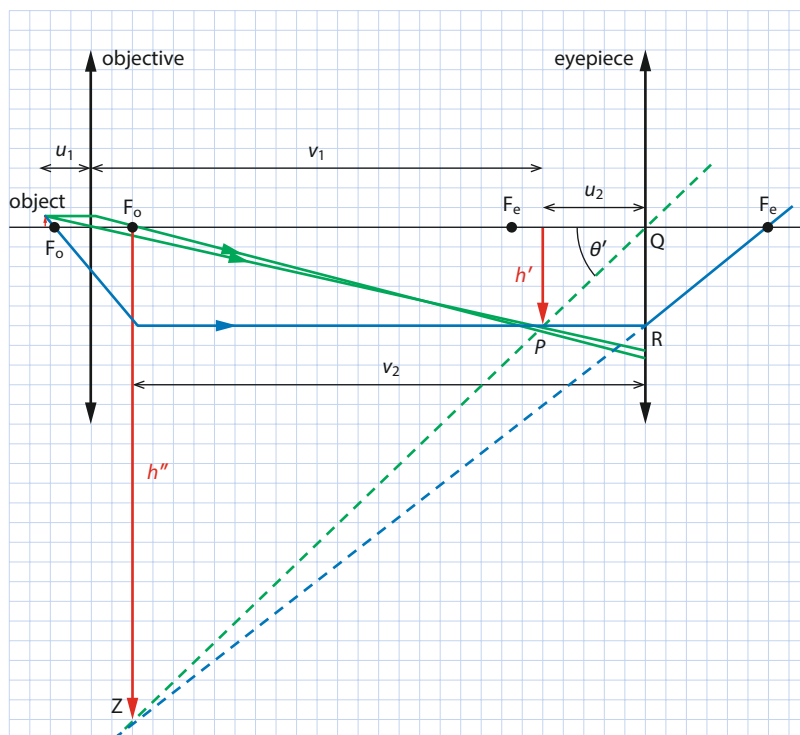


Figure C.34 A compound microscope consists of two converging lenses.

The diagram shows the intermediate image, determined by the three standard rays through the objective lens. One of the three standard rays, shown in dark blue, is also a standard ray for the eyepiece lens; from R it is refracted through the focal point of the eyepiece. To form the image, we draw the dashed green ray from P, the top of the intermediate image, to Q, the centre of the lens. Extended backwards, this intersects the extension of the dark blue ray at Z, the top of the final image.

The overall angular magnification of the microscope is defined, as usual, as the ratio of two angles: θ' , which the final image subtends at the eyepiece, to θ , which the original object would subtend when viewed from the near point distance D :

$$M = \frac{\theta'}{\theta}$$

But $\theta' \approx \frac{h''}{v_2}$ and $\theta \approx \frac{h}{D}$, so

$$M \approx \frac{\frac{h''}{v_2}}{\frac{h}{D}} \approx \frac{D}{f} = \frac{h'' D}{h v_2} = \frac{h''}{h'} \frac{h' D}{h v_2}$$

The overall angular magnification of the microscope is therefore

$$M \approx m_o \times m_e \times \frac{D}{v_2}$$

Exam tip

In a microscope we want an objective with a short focal length and an eyepiece with a long focal length.

Exam tip

General formula for angular magnification of a microscope.



where m_o and m_e are the **linear** magnifications of the objective and eyepiece lenses, respectively. The linear magnification of the objective is $m_o = -\frac{v_1}{u_1}$, where u_1 is the distance of the object and v_1 the distance of the image (in the objective) from the objective. The linear magnification of the eyepiece is $m_e = -\frac{v_2}{u_2}$. If the final image is formed at the near point (referred to as **normal adjustment**), then $v_2 = D$ and in that case $m_e = \frac{D}{f_e} + 1$ (see Section C1.7), so

$$M \approx m_o \times \left(\frac{D}{f_e} + 1 \right)$$

To understand the meaning of the angular magnification of a microscope, consider a microscope of overall angular magnification (-250) . Suppose that we are looking at an object that is $8\mu\text{m}$ long. This object, when magnified, will **appear** to have the same size as an object of size $250 \times 8\mu\text{m} = 2\text{mm}$ viewed from 25cm .

Exam tip

This is the angular magnification at normal adjustment of the microscope (i.e. when the image is at the near point).

Notice that, in this case, the angular and linear magnifications of the eyepiece lens are the same.

None of these formulas is in the data booklet; you will have to derive them.

Worked examples

C.8 A compound microscope has an objective of focal length 2.0 cm and an eyepiece of focal length 6.0 cm . A small object is placed 2.4 cm from the objective. The final image is formed 25 cm from the eyepiece. Calculate **a** the distance of the image in the objective from the objective lens, and **b** the distance of this image from the eyepiece lens. **c** Determine the overall magnification of the microscope.

a We use the lens formula for the objective to get

$$\frac{1}{2.4} + \frac{1}{v_1} = \frac{1}{2.0} \Rightarrow v_1 = 12\text{ cm}$$

b Now we do the same for the eyepiece to get

$$\frac{1}{u_2} + \frac{1}{-25} = \frac{1}{6.0} \Rightarrow u_2 = 4.8\text{ cm}$$

c The linear magnification of the objective is $m_o = -\frac{v_1}{u_1} = -\frac{12}{2.4} = -5.0$.

The angular magnification of the eyepiece is $M_e = 1 + \frac{D}{f_e} = 1 + \frac{24}{6.0} = 5.0$. The overall magnification is therefore $-5.0 \times 5.0 = -25$.

C.9 In a compound microscope the objective has a focal length of 1.0 cm and the eyepiece a focal length of 4.0 cm . A small object is placed 1.2 cm from the objective. The final image is formed 30 cm from the eyepiece. Calculate the magnification of the microscope.

Applying the formula $M_e \approx m_o \times m_e \times \frac{D}{v_2}$, we find

$$\frac{1}{1.2} + \frac{1}{v_1} = \frac{1}{1.0} \Rightarrow v_1 = 6.0\text{ cm}$$

Thus, $m_o = -\frac{6.0}{1.2} = -5.0$.

$$\frac{1}{u_2} + \frac{1}{-30} = \frac{1}{5.0} \Rightarrow u_2 = 4.29\text{ cm}$$

Thus, $m_e = -\frac{-30}{4.29} = 7.0$, so $M \approx -5.0 \times 7.0 \times \frac{25}{30} \approx 29$.

Exam tip

We would like to have a large angle α because the larger this angle the more light is collected by the microscope. So to reduce d_{\min} we have to put a medium of high refractive index between the object and the lens. Such microscopes are called oil immersion microscopes.

(You do not need to know this formula for the exam.)

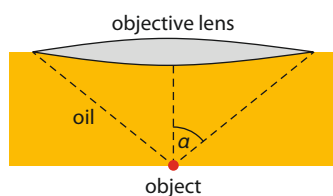


Figure C.35 An oil immersion microscope has a higher resolution than one without the oil.

Exam tip

In a telescope we want to have an objective with a long focal length and an eyepiece with a short focal length.

Exam tip

Formula for angular magnification of a refracting telescope with image at infinity (normal adjustment).

C2.2 Resolution of a compound microscope

Diffraction means that a point source will not have a point image in a lens; the image will be a disc of spread-out light. Thus if two point sources are very close to each other, their images will overlap and so may not be seen as distinct. This limits the resolution of the microscope. It can be shown that the smallest distance that can be resolved in a microscope is

$$d_{\min} = \frac{0.61\lambda}{n \sin \alpha}$$

where λ is the wavelength of the light and α is the angle shown in Figure C.35. There is a very small quantity of oil of refractive index n between the objective lens and the object. Since $n > 1$, this makes d_{\min} smaller than what it would be without the oil – that is, it increases the microscope's resolution.

C2.3 The refracting telescope

The function of a telescope is to allow the observation of large objects that are very distant and so appear very small. A **star** is enormous but looks small because it is far away. The telescope increases the angle subtended by the star relative to the angle subtended at the unaided eye. The telescope does not provide **linear** magnification of the star, since in that case the image would be many orders of magnitude larger than the Earth!

A refracting astronomical telescope (Figure C.36) consists of two converging lenses. Since the object observed is very far away, the image produced by the first lens (the objective) is at the focal plane of the objective. It is this image that is then magnified by the eyepiece, just as by a magnifying glass. The second lens (the eyepiece) forms a virtual, inverted image of the object. The final image is produced at infinity, so the distance between the two lenses is the sum of their focal lengths. Under these conditions the telescope is said to be in **normal adjustment**.

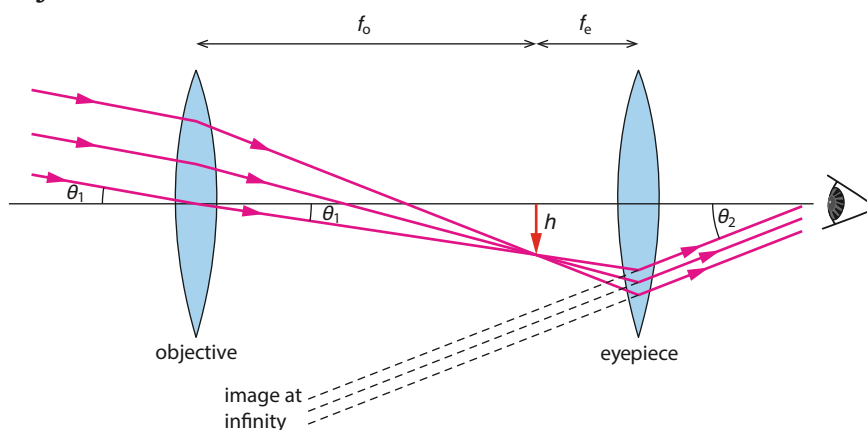


Figure C.36 A refracting astronomical telescope. The final image is inverted.

The **angular magnification** of the telescope is defined as the ratio of the angle subtended by the object as seen through the telescope to the angle subtended by it at the unaided eye. Thus:

$$M = \frac{\theta_2}{\theta_1} = \frac{h/f_e}{h/f_o} = \frac{f_o}{f_e}$$



The position of the eyepiece can be adjusted to provide clear images of objects other than very distant ones. The objective lens should be as large as possible in order to allow as much light as possible into the telescope. Because it is difficult to make very large lenses, telescopes have been designed to use mirrors rather than lenses.

Worked examples

C.10 A refracting telescope has a magnification of 70.0 and the two lenses are 60.0 cm apart at normal adjustment. Determine the focal lengths of the lenses.

The angular magnification is

$$M = \frac{f_o}{f_e} = 70 \Rightarrow f_o = 70f_e$$

so

$$f_o + f_e = 70f_e + f_e = 71f_e = 60 \text{ cm}$$

$$f_e = 0.845 \text{ cm}$$

$$f_o = 59.2 \text{ cm}$$

C.11 An astronomical telescope is used to view an object 20 m from the objective. The final real image is formed 30 cm from the eyepiece lens. The focal length of the objective is 4.0 m and that of the eyepiece is 0.80 m. Determine the overall linear magnification of the telescope.

The image in the objective is formed at a distance found from $\frac{1}{20} + \frac{1}{v_1} = \frac{1}{4.0} \Rightarrow v_1 = 5.0 \text{ m}$. Hence the linear magnification of the objective is $m_o = -\frac{5.0}{20} = -0.25$.

The object for the eyepiece is at a distance found from $\frac{1}{u_2} + \frac{1}{0.30} = \frac{1}{0.80} \Rightarrow u_2 = -0.48 \text{ m}$. Hence the linear magnification of the eyepiece is

$$m_e = -\frac{0.30}{-0.48} = 0.625.$$

The overall magnification is therefore $-0.25 \times 0.625 = -0.16$.

C2.4 Reflecting telescopes

Reflecting telescopes use mirrors rather than the lenses of **refracting telescopes**. This creates a number of advantages, including:

- To see distant faint objects requires large lenses (to collect more light). But large lenses are hard to make (the glass must be homogeneous and free of air bubbles); they can also only be supported along their rim, and large lenses may collapse under their own weight. By contrast, large mirrors can be supported along the rim and at the back.
- Mirrors do not suffer from chromatic aberration.
- Only one side has to be ground, as opposed to two for lenses.

For these reasons, the largest telescopes are reflecting.

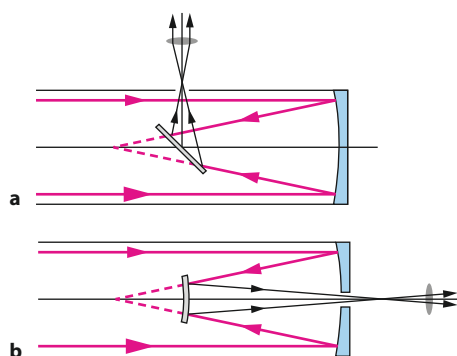


Figure C.37 Reflecting telescopes: **a** Newtonian; **b** Cassegrain.

Figure **C.37** shows two types of refracting telescope. In the first, known as **Newtonian**, light from a distant object is reflected from a parabolic mirror onto a smaller plane mirror at 45° to the axis of the telescope. The reflected light is collected by a converging lens which creates a parallel beam to the observer's eye. In the second type, known as the **Cassegrain** type, light is reflected from a parabolic mirror onto a much smaller convex mirror. Light reflecting off this mirror is collected by a converging lens that produces a parallel beam to the observer's eye.

C2.5 Single-dish radio telescopes

A **radio telescope** receives and detects electromagnetic waves in the radiofrequency region. Stars, galaxies and other objects are known to radiate in this region, so studying these emissions gives valuable information about the 'invisible' side of these objects. Recall from Topic **9** that diffraction places limits on resolution, that is, on the ability of an instrument to see two nearby objects as distinct. An instrument whose diameter is b and operates at a wavelength λ can resolve two objects whose angular separation (in radians) is θ_A if

$$\theta_A \geq 1.22 \frac{\lambda}{b}$$

Since radio wavelengths are large, the diameter of the radio telescope has to be large as well, in order to achieve reasonable resolution. The Arecibo radio telescope (Figure **C.38**) has a diameter of 300 m and operates at a wavelength of 21 cm. This means that it can resolve objects whose angular separation is no less than

$$\theta_A \approx \frac{1.22 \times 0.21}{300} \approx 8.5 \times 10^{-4} \text{ rad}$$

By contrast, an optical telescope such as the Hubble Space Telescope (HST) has a diameter of 2.4 m and operates at an average optical wavelength of 500 nm, so

$$\theta_A \approx \frac{1.22 \times 500 \times 10^{-9}}{2.4} \approx 2.5 \times 10^{-7} \text{ rad}$$

But large telescopes are very heavy steel structures, and difficult to steer. The Arecibo telescope is actually built into a valley and cannot be steered at all: it points to different parts of the sky only because the Earth rotates.

Radio telescopes have a parabolic shape. Parallel rays will therefore collect at the focus of the mirror, where a detector is placed.



Figure C.38 The Arecibo radio telescope in Puerto Rico.

C2.6 Radio interferometry telescopes

The low resolution of single-dish radio telescopes can be overcome by a technique known as **radio interferometry**. By using a very large array of radio telescopes very far apart and appropriately combining the signals from the individual dishes, one can achieve the same resolution as a single dish with a diameter equal to the length of the array. The Very Large Array (VLA) interferometer has 27 single dishes extending over 35 km (Figure C.39). It operates at a wavelength of 6 cm so its resolution is

$$\theta_A \approx \frac{1.22 \times 0.06}{35 \times 10^3} \approx 2.1 \times 10^{-6} \text{ rad}$$

This is only about 10 times lower than the HST.

C2.7 Satellite-borne telescopes

Earthbound telescopes are limited for a number of reasons, including:

- Light pollution (excess light in the atmosphere). This can be partly overcome by locating telescopes in remote areas, far from large cities.
- Atmospheric turbulence (mainly due to convection currents and temperature differences). This makes air move unpredictably, making the positions of stars appear to vary. It can be partly overcome by locating telescopes on high mountains, where the atmosphere tends to be more stable.
- Absorption of various wavelengths by the atmosphere. This makes observation at these wavelengths impossible. This is especially true for X-ray and ultraviolet wavelengths, which are almost completely absorbed by the atmosphere.

These problems do not exist for satellite-based telescopes in orbit around the Earth. The Hubble Space Telescope, shown in Figure C.40, a joint project of the European Space Agency (ESA) and the National Aeronautics and Space Administration (NASA), has truly revolutionised astronomy, and cosmology in particular, with its wealth of detailed images that have led to new discoveries and new areas of research.



Figure C.39 The Very Large Array in New Mexico, USA.



Figure C.40 The Hubble Space Telescope in orbit.

The spectacular image in Figure C.41 shows the supernova Cassiopeia A, a dying star. This image is a combination of images at different wavelengths: optical from the HST (yellow), infrared from the Spitzer space observatory (red) and X-rays from the Chandra X-ray observatory (blue).

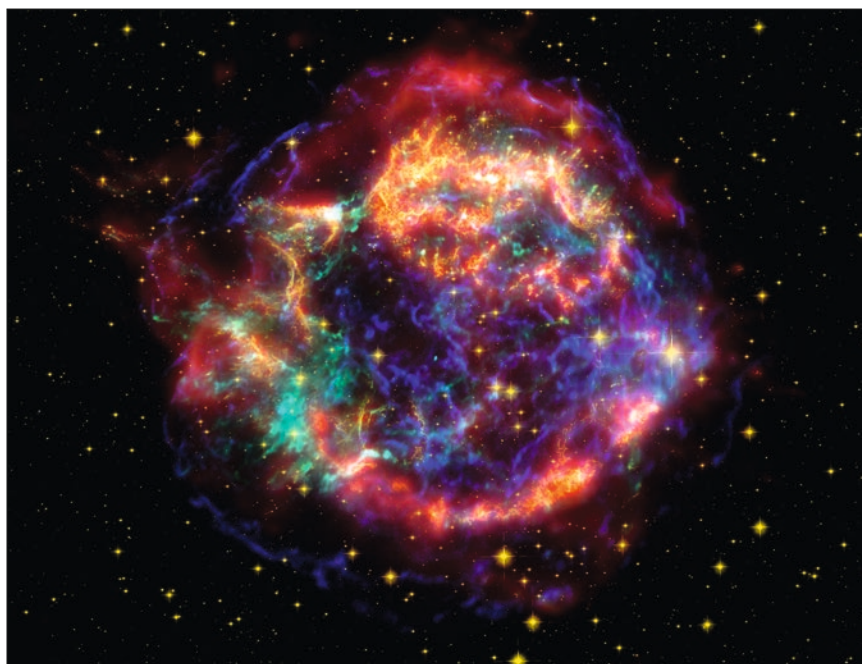


Figure C.41 A dying star, Cassiopeia A, after a supernova explosion more than 300 years ago.

Nature of science

Improved instrumentation

The photograph in Figure C.41 is an excellent example of the advances in imaging made by combining data from telescopes operating at different wavelengths. Observations that until recently were only made with optical telescopes on the Earth are now complemented by images from telescopes in space operating in the radio, infrared, ultraviolet, X-ray and gamma-ray regions of the electromagnetic spectrum. Placing telescopes away from the Earth's surface avoids the distorting effects of the Earth's atmosphere, and corrective optics enhances images obtained from observatories on the Earth. These developments have vastly increased our knowledge of the universe and have made possible discoveries and the development of theories about the structure of the universe that have exceeded even the most optimistic expectations. In exactly the same way, optical, electron and tunnelling microscopes have advanced our knowledge of the biological world, leading to spectacular advances in medicine and the treatment of disease.

? Test yourself



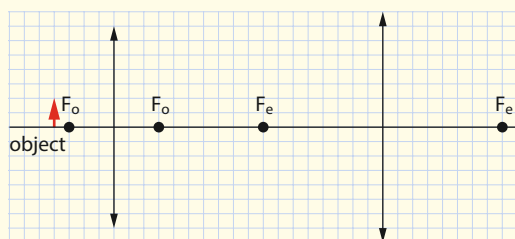
- 28 The objective of a microscope has a focal length of 0.80 cm and the eyepiece has a focal length of 4.0 cm. An object is placed 1.50 cm from the objective. The final image is formed at the near point of the eye (25 cm).

- Calculate the distance of the image from the objective.
- Calculate the distance from the eyepiece lens of the image in **a**.
- Calculate the angular magnification of the microscope.

- 29 In a compound microscope the objective focal length is 20 mm and the eyepiece focal length is 80 mm. An object is placed 25 mm from the objective. The final virtual image is formed 35 cm from the eyepiece.

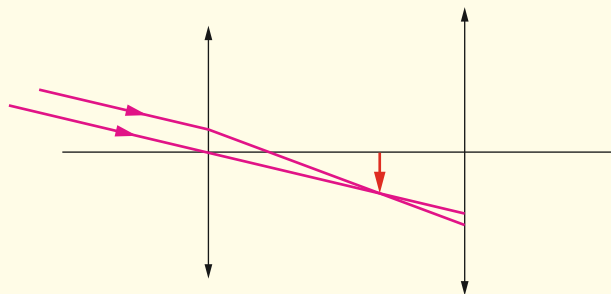
- Calculate the distance of the image from the objective.
- Calculate the distance from the eyepiece lens of the image in **a**.
- Calculate the angular magnification of the microscope.

- 30 The diagram below illustrates a compound microscope. Copy the diagram and draw rays in order to construct the final image.



- 31 A compound microscope forms the final image at a distance of 25 cm from the eyepiece. The eye is very close to the eyepiece. The objective focal length is 24 mm and the object is placed 30 mm from the objective. The angular magnification of the microscope is 30. Determine the focal length of the eyepiece.

- 32 The diagram below shows rays from a distant object arriving at a refracting telescope. Copy the diagram and complete the rays to show the formation of the final image at infinity.



- 33 An astronomical telescope is in normal adjustment.

- State what is meant by this statement.
- The angular magnification of the telescope is 14 and the focal length of the objective is 2.0 m. Calculate the focal length of the eyepiece.

- 34 The Moon is at a distance of 3.8×10^8 m from the Earth and its diameter is 3.5×10^6 m.

- Show that the angle subtended by the diameter of the Moon at the eye of an observer on the Earth is 0.0092 rad.
- A telescope objective lens has a focal length of 3.6 m and an eyepiece focal length of 0.12 m. Calculate the angular diameter of the image of the Moon formed by this telescope.

- 35 A telescope consists of an objective, which is a converging lens of focal length 80.0 cm, and the eyepiece of has a focal length 20.0 cm. The object is very far from the objective (effectively an infinite distance away) and the image is formed at infinity.

- Calculate the angular magnification of this telescope.
- The telescope is used to view a building of height 65.0 m a distance of 2.50 km away. Calculate the angular size of the final image.

- 36 A refracting telescope has an eyepiece of focal length 3.0 cm and an objective of focal length 67.0 cm.

- Calculate the magnification of the telescope.
- State the length of the telescope. (Assume that the final image is produced at infinity.)

- 37 A refracting telescope has a distance between the objective and the eyepiece of 60 cm. The focal length of the eyepiece is 3.0 cm. The eyepiece has to be moved 1.5 cm further from the objective to provide a clear image of an object some finite distance away. Estimate this distance. (Assume that the final image is produced at infinity.)
- 38 a State what is meant by radio interferometry.
b Estimate the resolution in radians of an array of radio telescopes extending over 25 km. The telescopes operate at a wavelength of 21 cm.
c Estimate the smallest separation that can be resolved by this array, in a galaxy that is 2×10^{22} m from the Earth.
- 39 Suggest why telescopes other than optical ones are in use.
- 40 Suggest why parabolic mirrors are used in telescopes.
- 41 State **two** advantages and **two** disadvantages of satellite-based telescopes.

Learning objectives

- Describe optical fibres and solve problems dealing with them.
- Describe the differences between step-index and graded-index fibres.
- Describe the difference between waveguide and material dispersion.
- Solve problems involving attenuation and the decibel scale.

C3 Fibre optics

This section introduces one very important channel of communication, the **optical fibre**. We discuss the optics of the optical fibre and the concept of the critical angle. We introduce two types of optical fibres, multimode and monomode fibres, and discuss these in the context of dispersion and attenuation.

C3.1 Total internal reflection and optical fibres

The phenomenon of **total internal reflection** was discussed in Topic 4. Here we summarise the main results. Figure C.42 shows a ray of light entering a medium of low refractive index from a medium of higher refractive index. The angle of refraction is greater than the angle of incidence. As the angle of incidence increase, the angle of refraction will eventually become 90° . The angle of incidence at which this happens is called the **critical angle**.

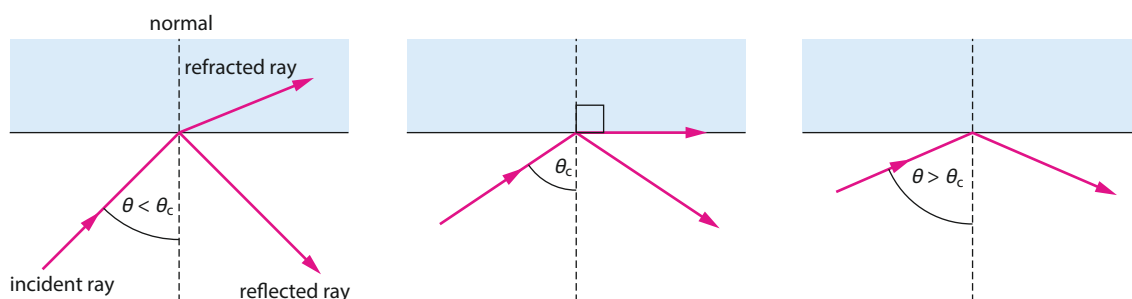


Figure C.42 A ray of light incident on a boundary partly reflects and partly refracts. The angle of refraction is larger than the angle of incidence.

The critical angle is the angle of incidence for which the angle of refraction is 90° .



The critical angle θ_c can be found from Snell's law:

$$n_1 \sin \theta_c = n_2 \sin 90^\circ$$

$$\sin \theta_c = \frac{n_2}{n_1}$$

$$\theta_c = \arcsin \frac{n_2}{n_1}$$

For an angle of incidence greater than the critical angle, no refraction takes place. The ray is simply reflected back into the medium from which it came. This is called **total internal reflection**.

One important application of total internal reflection is a device known as an **optical fibre**. This consists of a very thin glass core surrounded by a material of slightly lower refractive index (the **cladding**). Such a thin fibre can easily be bent without breaking, and a ray of light can be sent down the length of the fibre's core. For most angles of incidence, total internal reflection occurs (Figure C.43), so the light ray stays within the core and never enters the cladding.

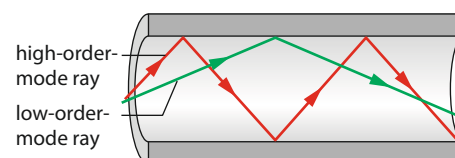


Figure C.43 A ray of light follows the shape of an optical fibre by repeated internal reflections.

Worked example

C.12 The refractive index of the core of an optical fibre is 1.50 and that of the cladding is 1.40. Calculate the critical angle at the core–cladding boundary.

From Snell's law, we have

$$1.50 \sin \theta_c = 1.40 \sin 90^\circ$$

$$\sin \theta_c = \frac{1.40}{1.50} = 0.9333$$

$$\theta_c = \arcsin 0.9333 = 69.0^\circ$$

C3.2 Dispersion

Because the refractive index of a medium depends on the wavelength of the light travelling through it, light of different wavelengths will travel through the glass core of an optical fibre at different speeds. This is known as **material dispersion**. Therefore, a set of light rays of different wavelengths will reach the end of a fibre at different times, even if they follow the same path.

Consider a pulse of light created by turning on, say, a light-emitting diode (LED) for a short interval of time. The power of the signal as a function of time as it enters the fibre is represented on the left-hand side of Figure C.44. The area under the pulse is the energy carried by the pulse. In the output pulse, on the right-hand side of Figure C.44, the area is somewhat smaller because some energy has been lost during transmission. Because of the different travel times, the pulse has become wider.

Rays of light entering an optical fibre will, in general, follow different paths. Rays that undergo very many internal reflections over a given distance are said to follow **high-order-mode** paths, while those suffering fewer reflections follow **low-order-mode** paths (Figure C.45).

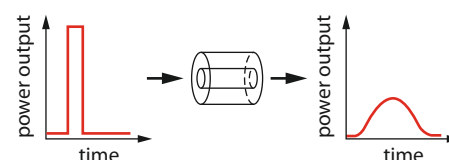


Figure C.44 The effect of material dispersion.

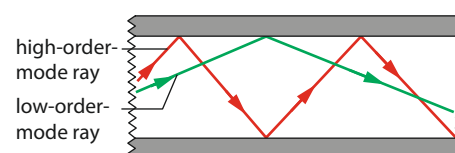


Figure C.45 Low-order- and high-order-mode rays in an optical fibre.

Consider a set of rays that have the same wavelength but follow different paths (i.e. they have different-order modes). Those rays travelling along low-order paths are more ‘straight’, travel a shorter distance, and so will reach the end faster than higher-order rays. This leads to what is called **waveguide dispersion**. The effect on the input signal of Figure C.44 is the same.

In practice, a set of rays will have different wavelengths and will follow different paths, so they will be subject to both material and waveguide dispersion. This is the case in **multimode** fibres (Figure C.46a and b). Multimode fibres have a core diameter of roughly $100\mu\text{m}$.

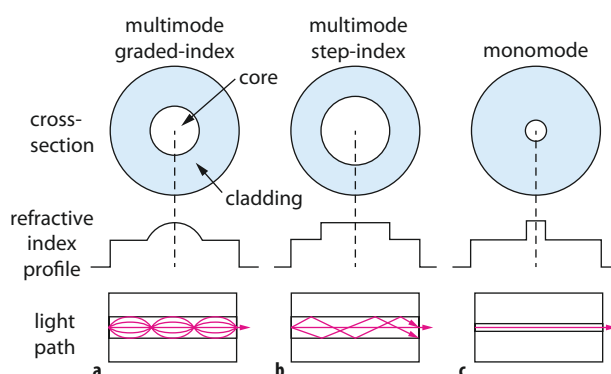


Figure C.46 **a** A multimode graded-index optical fibre. **b** A multimode step-index optical fibre. **c** A monomode optical fibre.

Of special interest are **monomode** fibres (Figure C.46c), in which all light propagates (approximately) along the same path. The diameter of the core of a monomode fibre is very small, about $10\mu\text{m}$, only a few times larger than the wavelength of the light entering it. The thickness of the cladding is correspondingly much larger, in order to make connecting one fibre to another easier. The propagation of light in such a fibre is not governed by the conventional laws of optics that we are using in this section. The full electromagnetic theory of light must be used, which results in the conclusion that light follows, essentially, just one path down the fibre, eliminating waveguide dispersion. Monomode fibres are now used for long-distance transmission of both analogue and digital signals.

Figure C.46b illustrates the meaning of the term **step-index fibre**. This means that the refractive index of the core is constant, and so is that of the cladding, but at a slightly lower value. The refractive index thus shows a ‘step’ (down) as we move from the core to the cladding. This type of fibre is to be contrasted with a **graded-index fibre**, in which the refractive index decreases smoothly from the centre of the core (where it reaches a maximum) to the outer edge of the core. The refractive index in the cladding is constant.

Exam tip

Graded index fibres help reduce waveguide dispersion: ordinarily, rays that move far from the central axis would take longer to arrive leading to dispersion; but in a graded-index fibre the speed of light away from the axis is also greater and so the longer path is covered at higher speed. The net effect is an almost constant arrival time independent of path.



Worked example

C.13 The length of an optical fibre is 5.0 km. The refractive index of the core of the optical fibre is 1.50 and the critical angle of the core–cladding boundary is 75° . Calculate the time taken for a ray of light to travel down the length of the fibre:

- along a straight line parallel to the axis of the fibre
- suffering the maximum number of internal reflections in the fibre.

The speed of light in the core of the fibre is determined by the refractive index:

$$c = \frac{3.00 \times 10^8}{1.50} = 2.00 \times 10^8 \text{ m s}^{-1}$$

- The distance travelled by the light in this case is 5.0 km, so the time taken is

$$t = \frac{5.0 \times 10^3}{2.00 \times 10^8} = 25 \mu\text{s}$$

- The ray must travel as shown in Figure C.47, with the angle θ_c being infinitesimally larger than 75° .

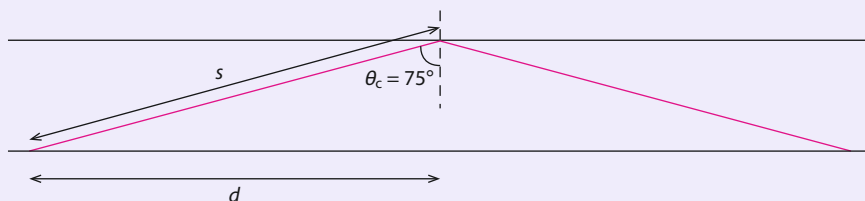


Figure C.47

Then $s = \frac{d}{\sin \theta}$. The total distance travelled by the ray is then $s = \frac{5.00}{\sin 75^\circ} = 5.18 \text{ km}$, and the time taken is

$$t = \frac{5.18 \times 10^3}{2.00 \times 10^8} = 26 \mu\text{s}.$$

C3.3 Attenuation

Any signal travelling through a medium will suffer a loss of power. This is called **attenuation**. It may be necessary to amplify the signal for further transmission. In the case of optical fibres, attenuation is mainly due to the scattering of light by glass molecules and impurities. The massive introduction of optical fibres into communications has been made possible by advances in the manufacture of very pure glass. For example, the glass in the window of a house appears to let light through without much absorption of energy, but a window pane is less than 1 cm thick. Glass of the same quality as that in ordinary windows and with a thickness of a few kilometres would not transmit any light at all.

Attenuation in an optical fibre is caused by the scattering of light and the impurities in the glass core. The amount of attenuation depends on the wavelength of light being transmitted.

To quantify attenuation, we use a logarithmic scale or **decibel scale**. We define the **power loss in decibels** (dB) as

$$\text{power loss (in dB)} = 10 \log \frac{P_{\text{final}}}{P_{\text{initial}}}$$

This is a negative quantity; P is the power of the signal.

Thus a power loss of 16 dB means that an initial power of, say, 8.0 mW has been reduced to a value given by

$$-16 = 10 \log \frac{P_{\text{final}}}{P_{\text{initial}}}$$

$$-1.6 = \log \frac{P_{\text{final}}}{8.0}$$

$$\frac{P_{\text{final}}}{8.0} = 10^{-1.6}$$

$$P_{\text{final}} = 8.0 \times 10^{-1.6} \approx 0.20 \text{ mW}$$

This idea can also be applied to signals that are amplified, as the next example shows.

Worked example

C.14 An amplifier amplifies an incoming signal of power 0.34 mW to 2.2 mW. Calculate the power **gain** of the amplifier in decibels.

The amplifier is shown schematically in Figure C.48.

For this amplifier, we have

$$\text{gain} = 10 \log \frac{P_{\text{out}}}{P_{\text{in}}} = 10 \log \frac{2.2}{0.34} = 10 \times 0.81 = 8.1 \text{ dB}$$

It is worth remembering that an increase in power by a factor of 2 results in a power gain of approximately 3 dB:

$$\text{gain} = 10 \log \frac{P_{\text{out}}}{P_{\text{in}}} = 10 \log 2 = 3.01 \approx 3 \text{ dB}$$

Similarly, a decrease in power by a factor of 2 implies a 3 dB power loss.

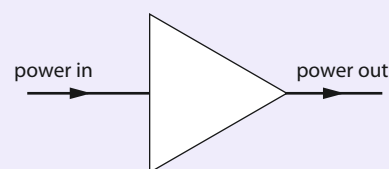


Figure C.48

Also useful is the concept of **specific attenuation**, the power loss in decibels per unit length travelled: $\text{specific attenuation} = \frac{10 \log P_{\text{out}}/P_{\text{in}}}{L}$. This is measured in decibels per kilometre (dB km⁻¹).

Worked examples

C.15 A signal of power 12 mW is input into a cable of specific attenuation 4.0 dB km⁻¹. Calculate the power of the signal after it has travelled 6.0 km in the cable.

The loss is $-4.0 \times 6.0 = -24 \text{ dB}$. Then

$$-24 = 10 \log \frac{P_{\text{out}}}{P_{\text{in}}}$$

$$-2.4 = \log \frac{P_{\text{out}}}{P_{\text{in}}}$$

$$\frac{P_{\text{out}}}{P_{\text{in}}} = 10^{-2.4} \approx 3.98 \times 10^{-3}$$

$$P_{\text{out}} = 3.98 \times 10^{-3} P_{\text{in}} = 3.98 \times 10^{-3} \times 12 = 0.048 \text{ mW}$$

C.16 A signal travels along a monomode fibre of specific attenuation 3.0 dB km^{-1} . The signal must be amplified when the power has decreased by a factor of 10^{18} . Calculate the distance at which the signal must be amplified.

We know that $\frac{P_{\text{out}}}{P_{\text{in}}} = 10^{-18}$. Therefore the loss is $10 \log \frac{P_{\text{out}}}{P_{\text{in}}}$. Hence we need amplification after $\frac{180 \text{ dB}}{3.0 \text{ dB km}^{-1}} = 60 \text{ km}$.

The specific attenuation (i.e. the power loss in dB per unit length) actually depends on the wavelength of the radiation travelling along the optical fibre. Figure C.49 is a plot of specific attenuation as a function of wavelength. The graph shows minima at wavelengths of 1310 nm and 1550 nm, which implies that these are desirable wavelengths for optimal transmission. These are infrared wavelengths.

C3.4 Advantages of optical fibres

In the early days of telephone communications, signals were carried by twisted pairs of wires (Figure C.50a).

As the name suggests, pairs of copper wires were twisted around each other. This reduces noise from induced currents caused by magnetic fields created by the currents in the wires. The twisting essentially has the current in the pair of wires going in opposite directions, thus limiting the value of the magnetic field. It does not, however, eliminate the problem of one pair affecting another. Many of the problems associated with twisted wires were solved by the introduction of the more reliable (and much more expensive) coaxial cable (Figure C.50b).

The optical fibre has a series of impressive advantages over the coaxial cable and, of course, twisted wires. These include:

- low attenuation
- no interference from stray electromagnetic signals
- greater capacity (bandwidth), making possible the transmission of very many signals
- security against 'tapping', i.e. unauthorised extraction of information from the signal.

Nature of science

Applied science

The development of optical fibres has been one of the main forces behind the revolution in communications that we experience today. The fast, clear and cheap transfer of information in digital form from one part of the world to another, with all that this implies about the free flow of information and immediate access to it, has a lot to do with the capabilities of modern optical fibres.

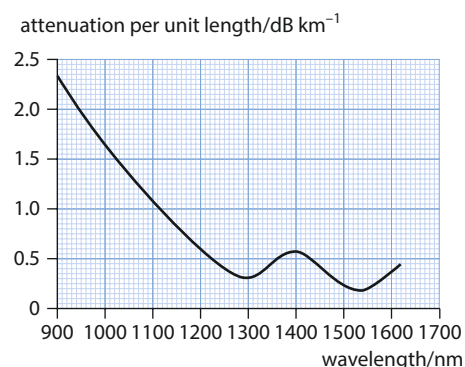


Figure C.49 The variation of specific attenuation with wavelength in a monomode fibre.

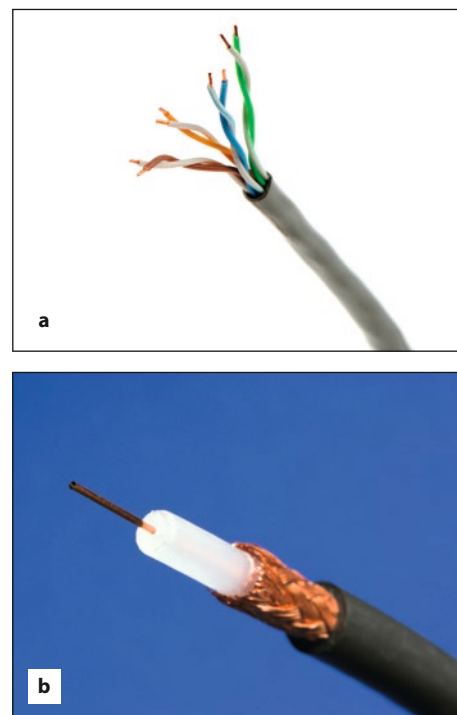
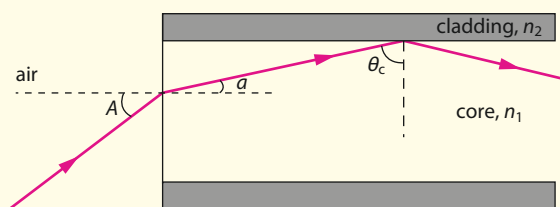


Figure C.50 **a** Twisted wire pairs; **b** coaxial cable.

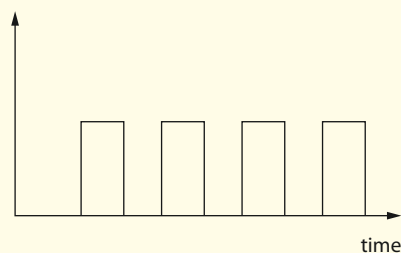
? Test yourself

- 42 Calculate the speed of light in the core of an optical fibre of refractive index 1.45.
- 43 **a** State what is meant by **total internal reflection**.
b Define **critical angle**.
c Explain why total internal reflection can only occur for a ray travelling from a high- to a low-refractive-index medium and not the other way around.
- 44 The refractive indices of the core and the cladding of an optical fibre are 1.50 and 1.46, respectively. Calculate the critical angle at the core-cladding boundary.
- 45 In an optical fibre, n_1 and n_2 are the refractive indices of the core and the cladding, respectively (so $n_1 > n_2$).

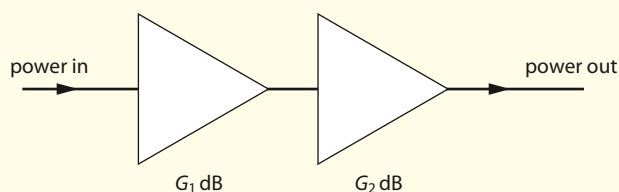


- a** Show that the cosine of the critical angle is given by
- $$\cos \theta_c = \frac{\sqrt{n_1^2 - n_2^2}}{n_1}$$
- b** Hence show that the maximum angle of incidence A from air into the core that will result in the ray being totally internally reflected is given by
- $$A = \arcsin \sqrt{n_1^2 - n_2^2}$$
- c** Calculate the acceptance angle of an optical fibre with a core refractive index of 1.50 and cladding refractive index of 1.40.
- 46 Calculate the acceptance angle of an optical fibre with core and cladding refractive indices equal to 1.52 and 1.44, respectively.
- 47 The refractive index of the cladding of an optical fibre is 1.42. Determine the refractive index of the core such that any ray entering the fibre gets totally internally reflected.
- 48 State **one** crucial property of the glass used in the core of an optical fibre.
- 49 **a** State what is meant by **dispersion** in the context of optical fibres.
b Distinguish between waveguide and material dispersion.

- 50 An optical fibre has a length of 8.00 km. The core of the optical fibre has a refractive index of 1.52 and the core-cladding critical angle is 82° .
a Calculate the speed of light in the core.
b Calculate the minimum and maximum times taken for a ray of light to travel down the length of the fibre.
- 51 The pulse shown below is input into a multimode optical fibre. Suggest the shape of the output pulse after it has travelled a long distance down the fibre.



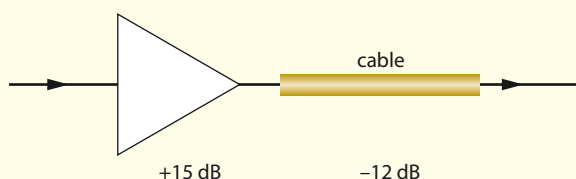
- 52 **a** Distinguish between **monomode** and **multimode** optical fibres.
b Discuss the effect of reducing the fibre core diameter on the bandwidth that can be transmitted by the fibre.
- 53 List **three** advantages of optical fibres in communications.
- 54 State the main cause of attenuation in an optical fibre.
- 55 Two amplifiers of gain G_1 and G_2 (in dB) amplify a signal, as shown below. Calculate the overall gain produced by the two amplifiers.



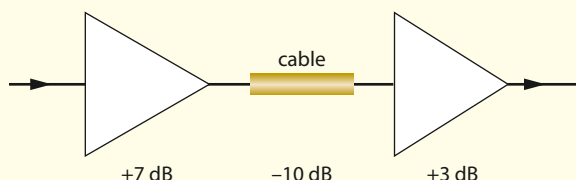
- 56 A signal of power 4.60 mW is attenuated to 3.20 mW. Calculate the power loss in decibels.
- 57 A signal of power 8.40 mW is attenuated to 5.10 mW after travelling 25 km in a cable. Calculate the attenuation per unit length of the cable.
- 58 A coaxial cable has a specific attenuation of 12 dB km^{-1} . The signal must be amplified when the power of the signal falls to 70% of the input power. Determine the distance after which the signal must be amplified.



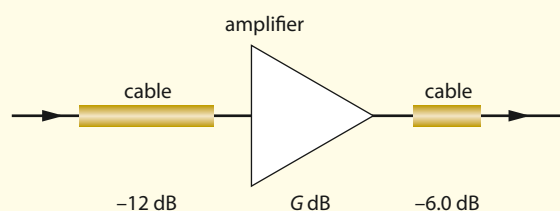
- 59 A signal is input into an amplifier of gain +15 dB. The signal then travels along a cable, where it suffers a power loss of 12 dB. Calculate the ratio of the output power to the input power.



- 60 A signal is input into an amplifier of gain +7.0 dB. The signal then travels along a cable, where it suffers a power loss of 10 dB, and is then amplified again by an amplifier of gain +3.0 dB. Calculate the ratio of the output power to the input power.



- 61 In the arrangement shown below, the output power is twice the input power. Calculate the required gain G of the amplifier.



- 62 a Sketch a graph (no numbers are required on the axes) to illustrate the variation with wavelength of the specific attenuation in an optical fibre.
b Explain why infrared wavelengths are preferred in optical fibre transmission.

C4 Medical imaging (HL)

This section introduces the use of X-rays and ultrasound in medical imaging. Other imaging techniques, including PET scans and a method based on nuclear magnetic resonance, are also discussed.

C4.1 X-ray imaging

X-rays are electromagnetic radiation with a wavelength around 10^{-10} m. X-rays for medical use are produced in X-ray tubes, in which electrons that have been accelerated to high energies by high potential differences collide with a metal target. As a result of the deceleration suffered by the electrons during the collisions and transitions between energy levels in the target atoms, X-rays are emitted (see Figure C.51). This was the first radiation to be used for medical imaging. Typical hospital X-ray machines operate at voltages of around 15–30 kV for a mammogram or 50–150 kV for a chest X-ray.

X-rays travelling through a medium suffer energy loss, referred to as **attenuation**. The dominant mechanism for this is the photoelectric effect: X-ray photons are absorbed by electrons in the medium and energy is transferred to the electrons. The effect is strongly dependent on the atomic number of the atoms of the medium. There is a substantial difference between the atomic numbers of the elements present in bone ($Z=14$) and soft tissue ($Z=7$), and bone absorbs X-rays more strongly than soft tissue. Hence, an X-ray image will show a **contrast** between bone and soft tissue.

Learning objectives

- Understand the use of X-rays in medical imaging.
- Understand the use of ultrasound in medical imaging.
- Understand magnetic resonance imaging in medicine.

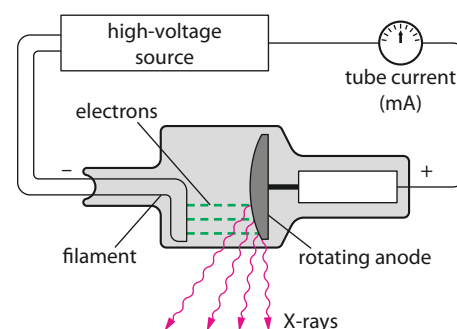


Figure C.51 Schematic diagram of an X-ray tube.

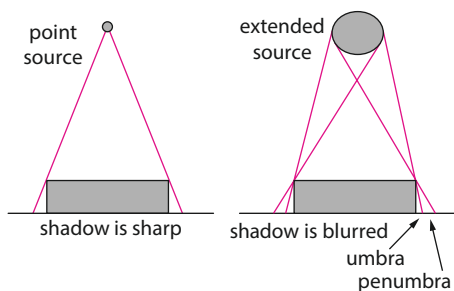


Figure C.52 The shadow cast is blurred if the source is extended.

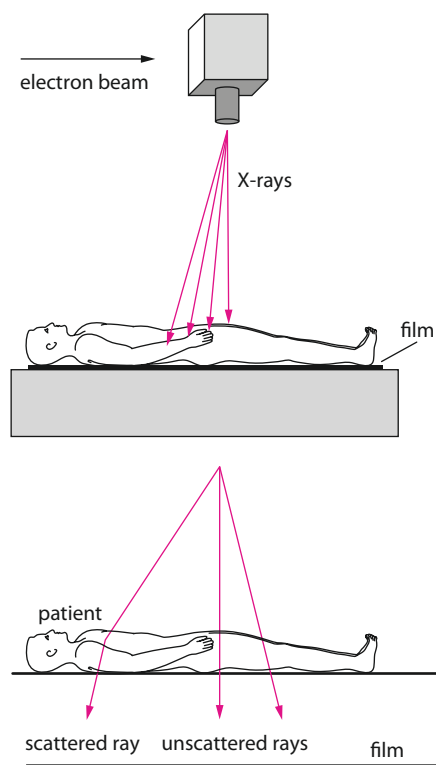


Figure C.53 A beam of X-rays entering a patient. Scattered rays blur the image.

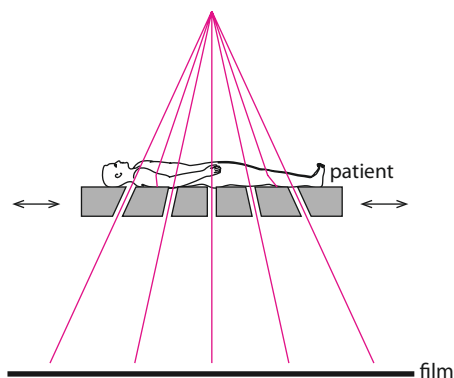


Figure C.54 The use of a grid of lead strips blocks scattered rays, improving the image.

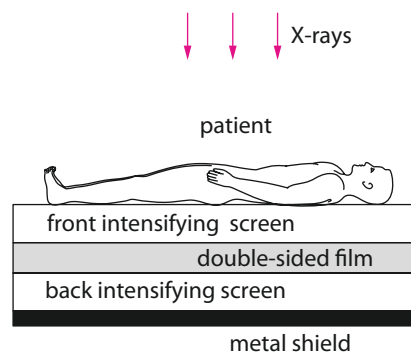


Figure C.55 The use of an intensifying screen increases the brightness of an X-ray image.

Where there is no substantial difference between the Z numbers of the area to be imaged and of the surrounding area – for example, in the digestive tract – the image can be improved by administering a **contrast medium**. Usually this consists of what is called a **barium meal** (barium sulfate), which the patient swallows. In the intestinal tract, the barium absorbs X-rays more strongly than surrounding tissue.

Those X-rays that pass through a patient's body fall on and expose photographic film. The image created by the X-rays on the film is thus a shadow of the high- Z material against surrounding low- Z tissue.

To increase the **sharpness** of the shadow, the X-ray source should be as point-like as possible (see Figure C.52). The quality of the image is thus improved if the film is as close to the patient as possible, or if the distance from the source to the patient is large. (In the latter case, the intensity of X-rays reaching the patient is diminished, which then requires a longer exposure time.)

The image is also improved if as many scattered rays as possible are prevented from reaching the film (Figure C.53). This can be achieved with the use of a grid of lead strips (lead readily absorbs X-rays) between the patient and the film, as shown in Figure C.54. The strips, about 0.5 mm apart, are closely oriented along the direction of the incoming X-rays, so scattered rays are absorbed. Unwanted images of the lead strips themselves can be minimised by moving the grid sideways back and forth during exposure so that the strip images are blurred.

Lower-energy X-rays tend to be absorbed by the patient's skin and are therefore of little use for imaging. These are usually filtered from the incoming beam.

Because photographic film is much more sensitive to visible light than to X-rays, the exposure time for an X-ray image must be longer. However, this can be significantly reduced by using an intensifying screen, containing fluorescent crystals on the front and rear surfaces and a double-sided photographic film in between. X-rays that have gone through the patient enter this screen and transfer some of their energy to the crystals. This energy is re-emitted as visible light and exposes the film (Figure C.55).

A technique called **fluoroscopy** allows for the creation of real-time, dynamic images. X-rays that have passed through the patient fall on a fluorescent screen and visible light is emitted. Directed at a photosurface, these photons cause the emission of electrons, which are accelerated through a potential difference and fall on a second fluorescent screen,

the light from which is fed into a TV monitor. The advantages of a real-time image may, however, be outweighed by the high doses of radiation that are needed.

C4.2 Computed tomography

A major advance in the medical use of X-rays was the development (in 1973) by G. N. Hounsfield and A. Cormack of a technique known as **computed (axial) tomography** (CT) or **computer-assisted tomography** (CAT). This has made possible much more accurate diagnosis using far less invasive procedures, though it does require the use of X-rays. A complete CAT brain scan lasts about 2 s and a whole-body scan about 6 s.

In a whole-body scan, a movable X-ray source emits a beam at right angles to the long axis of the patient, to be detected on the other side. The use of an array of detectors rather than just one reduces exposure levels and the time required for a scan. Figure C.56 shows a view from above the patient's head (the patient is represented by the grey circle). The source and detectors are rotated around the patient's body and moved along the length of the body. The data can then be combined into a three-dimensional computer image, viewable as two-dimensional 'slices' at any chosen position.

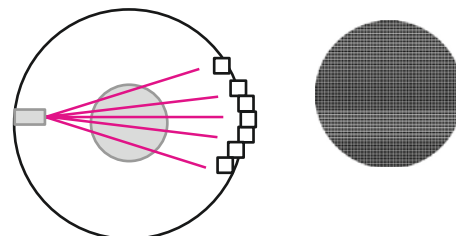


Figure C.56 In a CT scan, an array of detectors records the X-rays passing through the patient in many directions, allowing the construction of a three-dimensional image, which can then be viewed as 'slices' in any chosen direction and orientation.

C4.3 Attenuation

Imagine X-rays of intensity I_0 incident on a medium normally, as shown in Figure C.57. After travelling a distance x through the medium, the intensity of the X-rays has decreased to a value I , given by

$$I = I_0 e^{-\mu x}$$

Here μ is a constant called the **attenuation coefficient**. This coefficient can be determined from the slope of a plot of the logarithm of the intensity versus distance. It depends on the density ρ of the material through which the radiation passes, the atomic number Z of the material and the energy of the X-ray photons.

This relationship is similar to that for radioactive decay, and by analogy we define the **half-value thickness** (HVT), $x_{1/2}$, the penetration distance at which the intensity has been reduced by a factor of 2:

$$\frac{I_0}{2} = I_0 e^{-\mu x_{1/2}}$$

$$\frac{1}{2} = e^{-\mu x_{1/2}}$$

$$\ln \frac{1}{2} = -\mu x_{1/2}$$

Thus the half-value thickness and the attenuation coefficient are related by $\mu x_{1/2} = \ln 2$.

Figure C.58 shows the dependence on energy of the half-value thickness for X-rays and gamma rays in water.

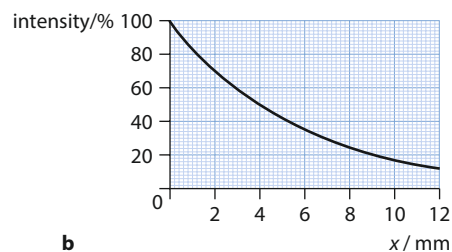
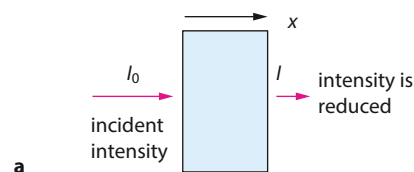


Figure C.57 **a** Attenuation of radiation in an absorbing medium. **b** The graph shows the intensity as a function of the penetration depth x .

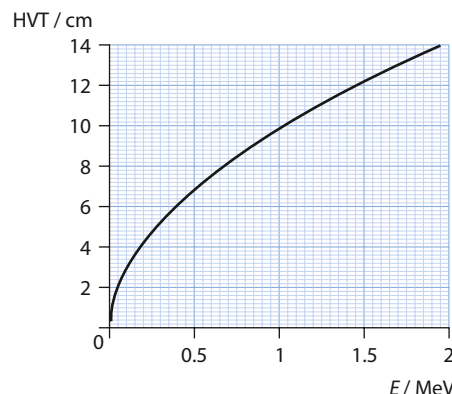


Figure C.58 Half-value thickness as a function of photon energy for X-rays.

Worked example

C.17 A metal sheet of thickness 4.0 mm and half-value thickness 3.0 mm is placed in the path of radiation from a source of X-rays. Calculate the fraction of the source's incident intensity that gets transmitted through the sheet.

We use $I = I_0 e^{-\mu x}$. First we have to find the attenuation coefficient μ . From $\mu x_{\frac{1}{2}} = \ln 2$ we find that

$$\mu = \frac{\ln 2}{x_{\frac{1}{2}}} = \frac{\ln 2}{3.0} = 0.231 \text{ mm}^{-1}.$$

Therefore

$$\begin{aligned} I &= I_0 e^{-\mu x} \\ &= I_0 \exp(-0.231 \times 4.0) \\ &= 0.397 I_0 \end{aligned}$$

or about 40% of the intensity goes through.

Because, for a given atomic number Z and energy E , the attenuation coefficient is proportional to the density, we can define a new coefficient, the **mass absorption coefficient**:

$$\mu_m = \frac{\mu}{\rho}$$

This allows comparisons, for given Z and E , between the attenuation in materials with different densities.

C4.4 Ultrasound

Ultrasound is a major tool in diagnostic medicine. The term refers to sound that is inaudible to the human ear, with a frequency higher than about 20 kHz. The ultrasound used in diagnostic medicine is in the range of about 1–10 MHz. Ultrasound has the advantage over X-rays that it does not deposit radiation damage in the body, and no adverse side effects of its use are known. A disadvantage of ultrasound is that the images are not as detailed as those from X-rays.

The ultrasound is emitted towards the patient's body in short pulses, typically lasting 1 μs , and their reflections off the surfaces of various organs are detected. The idea is thus similar to sonar. A 1 μs pulse of 1 MHz sound contains a single wavelength, while a 1 μs pulse of 10 MHz sound contains 10 wavelengths. The speed of sound in soft tissue is about 1540 m s^{-1} , similar to that in water, giving a wavelength of about 1.54 mm for 1 MHz ultrasound and 0.154 mm for 10 MHz ultrasound.

In general, diffraction places a limit on the size d that can be resolved using a wavelength λ . The constraint is that

$$\lambda < d$$

If resolution of a few millimetres is required, the wavelength used must therefore be less than this. Since 10 MHz ultrasound has a wavelength of about 0.15 mm, in principle it can 'see' objects of this linear size or

resolve objects with this separation. In practice, however, the pulse must contain at least a few full wavelengths for resolution at this level.



For the ultrasound frequencies used in medicine, it is the pulse duration, and not diffraction, that sets the limit on resolution.

The frequency to be used is usually determined by the organ to be studied and the resolution desired. A rough rule of thumb is to use a frequency of $f = 200 \frac{c}{d}$, where c is the speed of sound in tissue and d is the depth of the organ below the body surface. (Thus, for a given frequency, the organ should be at a depth of no more than about 200 wavelengths.)

Worked example

C.18 The stomach is about 10 cm from the body's surface. Suggest what frequency should be used for an ultrasound scan of the stomach.

Applying the formula gives

$$\begin{aligned} f &= 200 \frac{c}{d} \\ &= 200 \times \frac{1548}{0.10} \text{ Hz} \\ &\approx 3 \text{ MHz} \end{aligned}$$

The source of the ultrasound is a **transducer** that converts electrical energy into sound energy, using the phenomenon of **piezoelectricity**. An alternating voltage applied to opposite faces of a crystal such as strontium titanate or quartz will force the crystal to vibrate, emitting ultrasound (see Figure C.59). Likewise, ultrasound falling on such a crystal will produce an alternating voltage at the faces of the crystal. This means that the source of ultrasound can also act as the receiver.

The sound energy must then be directed into the patient's body. In general, when a wave encounters an interface between two different media, part of the wave will be reflected and part will be transmitted. The degree of transmission depends on the **acoustic impedances** of the two media. Acoustic impedance is defined as

$$Z = \rho c$$

where ρ is the density of the medium and c is the speed of sound in that medium. The units of impedance are $\text{kg m}^{-2} \text{ s}^{-1}$. If I_0 is the incident intensity, I_t the transmitted intensity and I_r the reflected intensity,

$$\begin{aligned} \frac{I_t}{I_0} &= \frac{4Z_1 Z_2}{(Z_1 + Z_2)^2} \\ \frac{I_r}{I_0} &= \frac{(Z_1 - Z_2)^2}{(Z_1 + Z_2)^2} \end{aligned}$$

This shows that for most of the energy to be transmitted, the impedances of the two media must be as close to each other as possible (**impedance matching**). The impedance of soft tissue differs from that of air by a factor of about 10^4 , so most ultrasound would be reflected at an air–tissue

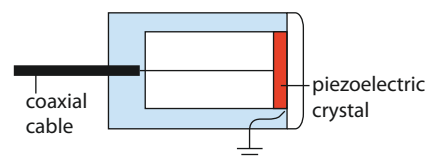


Figure C.59 A piezoelectric crystal vibrates when an ac source is applied to it.

interface. This is why the space between the body and the transducer is filled with a gel-like substance whose impedance closely matches that of soft tissue.

Worked example

C.19 The density of air is 1.2 kg m^{-3} and the speed of sound is 340 m s^{-1} . The density of soft tissue is about 1200 kg m^{-3} and the speed of sound in soft tissue is about 1500 m s^{-1} .

- Calculate the impedances of air and of soft tissue.
- Calculate the fraction of the intensity of ultrasound that would be transmitted from air into soft tissue.
- Comment on your answer.

a The impedances are given by $Z = \rho c$, so $Z_{\text{air}} = 1.2 \times 340 = 408 \text{ kg m}^{-2} \text{ s}^{-1}$ and $Z_{\text{tissue}} = 1200 \times 1500 = 1.8 \times 10^6 \text{ kg m}^{-2} \text{ s}^{-1}$.

b The transmitted fraction is $\frac{I_t}{I_0} = \frac{4Z_{\text{air}}Z_{\text{tissue}}}{(Z_{\text{air}} + Z_{\text{tissue}})^2} = \frac{4 \times 408 \times 1.8 \times 10^6}{(408 + 1.8 \times 10^6)^2} \approx 9 \times 10^{-4}$

c This is a negligible amount: most of the ultrasound is reflected. This shows the need to place a suitable gel between the transducer and the skin.

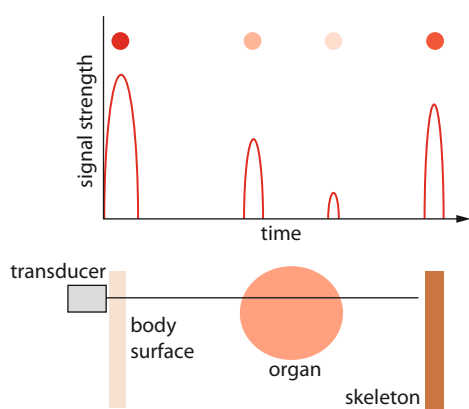


Figure C.60 A beam of ultrasound is directed into the patient. The beam is partially reflected from various organs in the body and the reflected signal is recorded. The dots at the top of the figure represent the strength of the reflected signal.

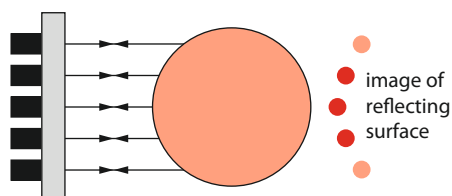


Figure C.61 As the source of ultrasound is moved across the body, a series of dots – whose brightness represents the strength of the reflected signal – is formed. These dots build up an ultrasound image.

In a type of ultrasound scan called an **A scan**, a pulse is directed into the body and the reflected pulses from various interfaces in the body are recorded by the transducer. These signals are converted into electrical energy and the reflected signal is displayed on a cathode-ray oscilloscope (CRO). The CRO signal is, in fact, a graph of signal strength versus time of travel from the transducer to the reflecting surface and back. An example of such a trace is shown in Figure C.60.

The dots in the graph show another way of representing the results, with dot brightness proportional to the signal strength. The A scan provides a one-dimensional image.

Now imagine a series of A scans using parallel beams of ultrasound with a transducer that moves along the body surface or with a series of transducers, as shown in Figure C.61. If these A scans are assembled, the result is a pattern such as the dots on the right of the diagram, which form a two-dimensional image of the surface of the organ.

Now imagine using a series of transducers, each sending one short pulse after the other, with a typical time delay of 1 ms. If these signals are displayed on the CRO screen, the result is a time-varying two-dimensional image of the organ – effectively a real-time ultrasound video. This is called a **B scan**.

Ultrasound can also be put to other uses, including observation of fetal heart movements and measurement of blood-flow velocities. Ultrasound directed at a moving organ or other object gives a reflected signal which is Doppler-shifted (its frequency is shifted). Comparison of the emitted and received frequencies reveals the speed of motion of the reflecting surface.

C4.5 Magnetic resonance imaging

Magnetic resonance imaging (MRI) is based on a phenomenon known as **nuclear magnetic resonance**, and is in many ways superior to CT scans. Unlike CT, the image is constructed without the use of dangerous radiation (despite the term ‘nuclear’), though the procedure is significantly more expensive.

Electrons, protons and some other particles have a property called **spin**. A particle with electric charge and spin behaves like a microscopic magnet (the technical term is **magnetic moment**). In the presence of an external magnetic field, the magnetic moment will align itself either parallel ('spin-up') or anti-parallel ('spin-down') to the direction of the magnetic field. Protons, for example, will have only certain energy values, depending on how their magnetic moments are aligned in the field (see Figure C.62), and the energy difference is proportional to the size of the field.

Exam tip

For exam purposes, it may be helpful to remember the main points of MRI in bullet form:

- A proton aligns itself parallel or anti-parallel to a strong external field.
- A radiofrequency signal forces the proton to change from a spin-up to a spin-down state.
- The proton then returns to the spin-up state, in the process emitting a photon of the same frequency.
- This frequency depends on the energy difference between the states, and therefore on the magnetic field at the proton's position.
- In a region with a different external field, a different radiofrequency will be needed to excite proton transitions.
- A secondary, non-uniform magnetic field is applied so that different parts of the body are exposed to different net magnetic fields.
- Each part of the body is then revealed by a different frequency of emitted photons.
- The rate of these transitions also gives information about tissue type.

The spin-up state has lower energy. If radiofrequency (RF) electromagnetic radiation provides energy to a sample of hydrogen nuclei (i.e. protons) in a magnetic field, those in the spin-up state may absorb photons and make a transition to the higher-energy spin-down state. This will happen if the radiation frequency matches the energy difference between the spin-up and spin-down states (an example of **resonance**). The transition up is followed by a transition down again, with the emission of another photon of the same frequency. Detectors record these photons, and techniques similar to CT scanning are used to create an image; detected photons can be correlated with specific points of emission.

The patient lies in an enclosure surrounded by a powerful magnet which creates a uniform magnetic field (Figure C.63). An additional,

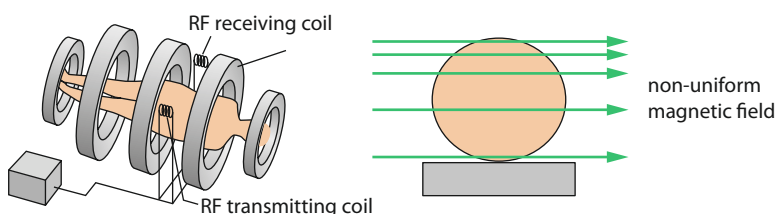


Figure C.63 In MRI, the patient is surrounded by powerful magnets that produce very uniform fields, and by RF coils. The magnetic field is then given a gradient (it is made stronger in some places). Here the field increases as we move upwards.

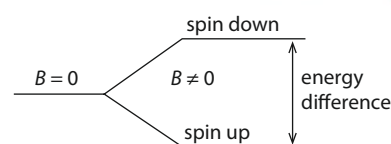


Figure C.62 A normal proton energy level splits in two in an external magnetic field. The spin-up state has a lower energy than the spin-down state.

non-uniform magnetic **gradient field** is superimposed on this, so that the total field varies across the patient (shown in cross-section in the figure). Imagine the variation of the magnetic field to be the same in parallel horizontal planes through the patient.

Now, a magnetic field which is different in different parts of the body will also mean different frequencies of resonant photon absorption and re-emission. Thus, for a particular RF frequency, only one plane within the body will have the correct value of magnetic field for this to take place. Other RF frequencies will give rise to resonance in other parts of the body with different magnetic fields. Variation of the RF frequency will therefore produce a series of photon patterns that can be combined, as with CAT, into a three-dimensional computer image.

More sophisticated techniques also measure the **proton spin relaxation** time – the rate at which excited protons return to their lower states – and these produce images of especially high resolution. Different types of tissue show different relaxation times, thus allowing the identification of particular types of tissue.

The various imaging techniques described in this section are summarised in Table C.1.

| Method | Resolution | Advantages | Disadvantages |
|------------|------------|--|--|
| X-ray | 0.5 mm | Inexpensive | Radiation danger Some organs are not accessible Some images are obscured |
| CT scan | 0.5 mm | Can distinguish between different types of tissue | Radiation danger |
| MRI | 1 mm | No radiation dangers Non-invasive Superior images Can distinguish between different types of tissue | Expensive and bulky equipment Difficult for patients who are claustrophobic Long exposures Exposure to magnetic fields difficult for patients with pacemakers and metallic hip implants |
| Ultrasound | 2 mm | No radiation dangers Versatile | Some organs are not accessible |

Table C.1 Advantages and disadvantages of various imaging techniques.

Nature of science

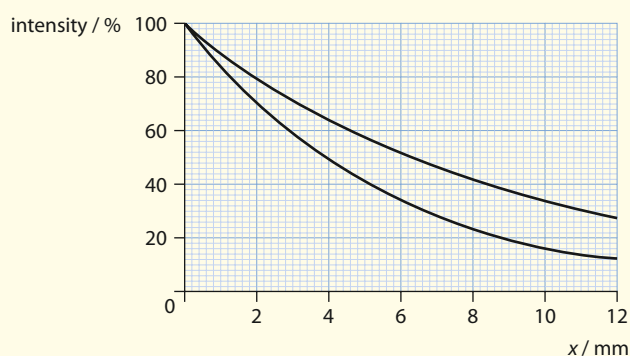
Risk analysis

In many diagnostic imaging techniques, a radiation danger is presented to the patient. That risk must be balanced by the complications to a patient's health should an existing serious condition go unnoticed by not doing the imaging tests. The risk of developing cancer as a result of the radiation dose received in a particular imaging technique depends on the age and gender of the patient, their BMI (body mass index – as a larger dose is needed for imaging thicker body tissues), and the part of the body being investigated. The risks can only be calculated as a probability, based on available evidence. It is believed that there is no completely safe lower limit, and risk increases in proportion to the dose. The doctor's role is to choose the imaging technique that minimises the risk to the patient and provides the greatest overall benefit.

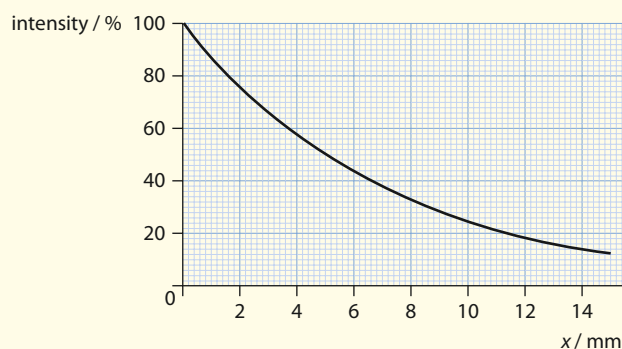
? Test yourself



- 63 a State what is meant by **attenuation**.
 b Describe how X-rays in a medium get attenuated.
- 64 Describe how the exposure time to X-rays during imaging may be reduced.
- 65 Describe the use of contrast media in X-ray imaging.
- 66 X-rays images may be blurry even though the patient is not moving. Suggest how this may be reduced.
- 67 The graph shows the fractions of X-rays of two specific energies transmitted through a thickness x of a sheet of metal.



- a For each energy, determine the half-value thickness for these X-rays in the metal.
- b Determine which graph corresponds to X-rays of higher energy.
- 68 The graph shows the fraction of X-rays of a specific energy transmitted through a thickness x of a sheet of metal.



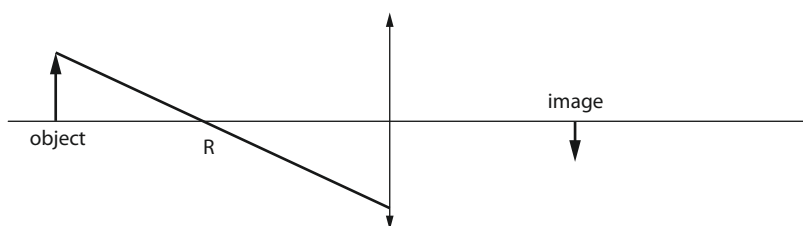
- a Determine the value of the attenuation coefficient for these X-rays in this metal.
- b Determine the thickness of metal that is required to reduce the transmitted intensity by 80%.

- 69 A piece of metal 4.0 mm thick reduces the intensity of X-rays passing through it by 40%. What thickness of the same metal is required to reduce the intensity by 80%?
- 70 The half-value thickness for a beam of X-rays in a particular metal is 3.0 mm. Determine the fraction of X-ray intensity that is transmitted through 1.0 mm of this metal.
- 71 The X-rays used in medicine are usually not monoenergetic (i.e. of a single energy). It is said that these beams become 'harder' as they are allowed to pass through a material.
- a Suggest what is meant by this statement and why it is true.
- b The half-value thickness of a certain absorber for X-rays of energy 20 keV is 2.2 mm and that for 25 keV X-rays is 2.8 mm. A beam containing equal quantities of X-rays of these two energies is incident on 5.0 mm of the absorber. Calculate the ratio of 25 keV to 20 keV photons that are transmitted.
- 72 State what is meant by **ultrasound** and describe how it is produced.
- 73 Estimate the resolution that can be achieved with ultrasound of frequency 5 MHz. (Take the speed of sound in soft tissue to be 1540 m s^{-1} .)
- 74 a State what is meant by **impedance**.
 b The density of muscle is 940 kg m^{-3} and the impedance of muscle is $1.4 \times 10^6 \text{ kg m}^{-2} \text{ s}^{-1}$. Calculate the speed of sound in muscle.
- 75 Ultrasound is directed from air into a type of tissue. The impedance of air is $420 \text{ kg m}^{-2} \text{ s}^{-1}$ and that of tissue is $1.6 \times 10^6 \text{ kg m}^{-2} \text{ s}^{-1}$.
- a Calculate the fraction of the incident intensity that gets transmitted into the tissue.
- b Comment on your answer.
- 76 Distinguish between **A scans** and **B scans** in ultrasound imaging.
- 77 Suggest the role of the gradient magnetic field in MRI.
- 78 Describe the method of **magnetic resonance imaging**.

Exam-style questions

- 1 a State what is meant by the **focal length** of a converging lens. [2]
- b In order to view the detail on an ancient coin, an art dealer holds a converging lens 2.0 cm above the coin. A virtual upright image of the coin is formed with a magnification of 5.0. Calculate the focal length of the lens [3]
- c Determine where the object should be placed so that the magnification produced is as large as possible. [3]

- 2 The diagram below shows an object placed in front of a converging lens. The lens forms an image of the object. The diagram also shows a ray R from the object.



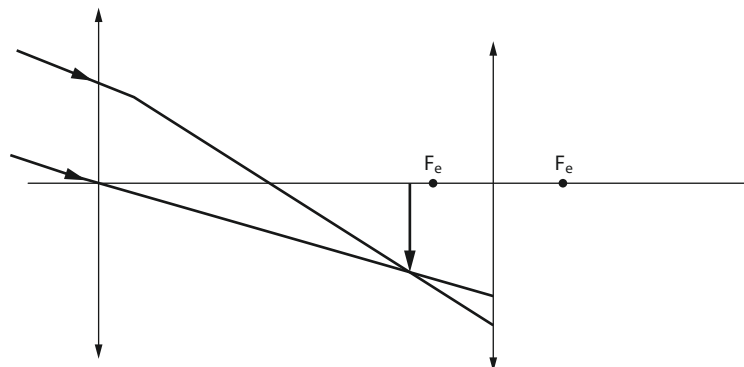
- a On a copy of the diagram, extend the ray R to show how it refracts in the lens. [1]
 - b Draw an appropriate ray to locate the focal points of the lens. [2]
 - c State and explain whether the image formed is real or virtual. [2]
 - d The upper half of the lens is covered with opaque paper. State and explain the effect of this, if any, on the image. [2]
 - e A converging lens of focal length 4.0 cm is used as a magnifying glass in order to view an object of length 5.0 mm placed at right angles to the axis of the lens. The image is formed 25 cm from the eye, which is placed very close to the lens. Determine:
 - i the distance of the object from the lens [2]
 - ii the length of the image [2]
 - iii the angle subtended by the image at the eye. [1]
- 3 A compound microscope consists of an objective lens of focal length 15 mm and an eyepiece lens of focal length 60 mm. The final image of an object placed 20 mm from the objective is formed 25 cm from the eyepiece lens.
 - a Determine:
 - i the distance of the image formed by the objective from the objective lens [2]
 - ii the distance of the image in i from the eyepiece lens. [2]
 - b i State what is meant by the **angular magnification** of a microscope. [2]
 ii Determine the angular magnification of the microscope. [2]
 - c The object has a length of 8.0 mm and is placed at right angles to the axis of the microscope. Calculate:
 - i the length of the final image [1]
 - ii the angle subtended by the final image at the eyepiece lens. [1]



4 An astronomical refracting telescope consists of two converging lenses.

a Suggest a reason why the diameter of the objective lens of a telescope should be large. [1]

b An astronomical telescope is used to view the Sun. The diagram below (not to scale) shows the formation of the intermediate image of the Sun. On a copy of the diagram, draw lines to show the formation of the real image of the Sun in the eyepiece. [2]



c The focal length of the objective is 1.00 m and that of the eyepiece is 0.10 m. Calculate the distance of the image in b from the eyepiece, given that the eyepiece forms a **real** image 0.455 m from the eyepiece. [2]

d The rays of the Sun make an angle of 0.055 rad with the axis of the objective. Determine the size of the image in the eyepiece. [3]

5 a An object of height 3.0 cm is placed 8.0 cm in front of a concave spherical mirror of focal length 24 cm. Calculate:

i the position of the image [1]

ii the height of the image. [2]

b Draw a ray diagram to illustrate your answers to i and ii. [3]

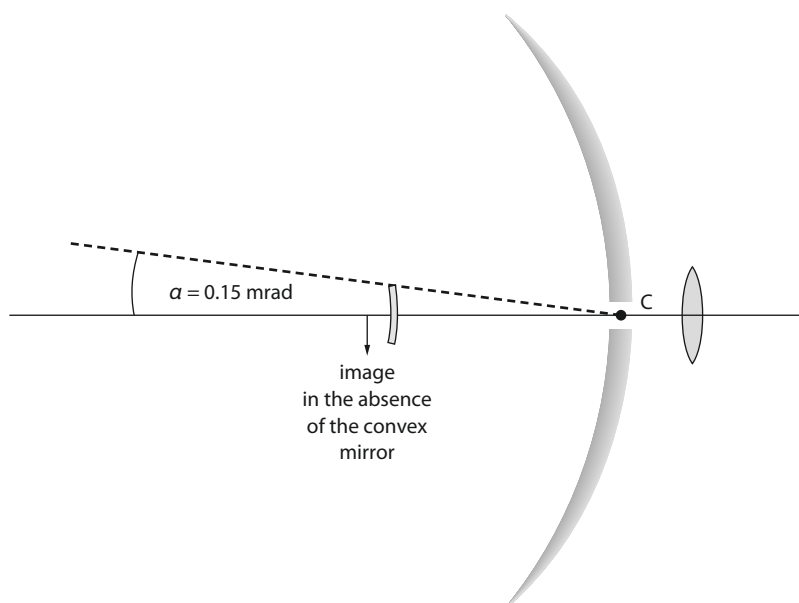
c An object is placed in front of a mirror. An upright image half the height of the object is formed behind the mirror. The distance between the object and the image is 120 cm. Calculate the focal length of the mirror. [3]

d Telescopes use mirrors rather than lenses.

i Outline **two** advantages of mirrors over lenses in a telescope. [4]

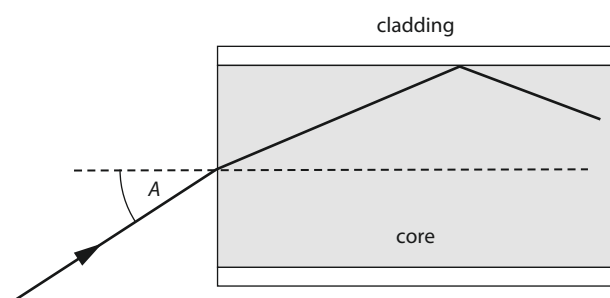
ii State **one** advantage of parabolic mirrors over spherical mirrors in a telescope. [1]

- 6 The diagram below (not to scale) shows a Cassegrain-type reflecting telescope. The small arrow shows the image of a planetary feature that would be formed by the concave mirror in the absence of the small convex mirror. The actual feature subtends an angle $\alpha = 1.50 \times 10^{-4}$ rad at the concave mirror. The focal length of the concave mirror is 10.0 m.



- a Calculate the length of the image shown here. [2]
- b This image serves in turn as the object for the small convex mirror, which produces a real image at C. The concave and convex mirrors are separated by 9.00 m. Calculate:
- i the focal length of the convex mirror [2]
 - ii the magnification of the convex mirror [1]
 - iii the height of the image at C. [1]
- c The image at C is viewed through a converging lens of focal length 12 cm, forming a virtual image very far away.
- i Calculate the angle subtended at the converging lens by the image at C. [2]
 - ii Hence calculate the overall angular magnification of this system. [2]
- 7 a State what is meant by a **lens aberration**. [1]
- b i **Spherical** and **chromatic aberration** are two common types of lens aberration. Describe what is meant by each. [4]
- ii Describe **one** way in which each of the aberrations in i may be reduced. [2]
- 8 Two objects, A and B, each of height 5.0 cm, are placed in front of a concave mirror of focal length 24 cm. The distances of the objects A and B from the mirror are, respectively, 40 cm and 30 cm.
- a Calculate:
- i the distance between the images of A and B [3]
 - ii the difference in heights of the images of A and B. [3]
- b A rod of length 10 cm is placed in front of the concave mirror such that it is parallel to the principal axis of the mirror and 5.0 cm to the side of it. The front of the rod is 30 cm from the mirror. Use your answer in a to determine whether the image of the rod:
- i has the same length as the rod itself [2]
 - ii is parallel to the principal axis. [2]

- 9 The diagram shows a ray of light entering the core of an optical fibre from air. The core has a refractive index of 1.58 and the cladding a refractive index of 1.45.



- a Determine:
- i the critical angle at the core–cladding boundary [2]
 - ii the largest angle of incidence A such that the ray will undergo total internal reflection at the core–cladding boundary. [2]
- b Distinguish between **waveguide dispersion** and **material dispersion** in an optical fibre. [3]
- c Outline how i waveguide dispersion and ii material dispersion may be reduced. [4]
- d The power of a signal input into an optical fibre is 25.0 mW. The attenuation per unit length for this fibre is 3.50 dB km⁻¹. The signal power must not fall below 15.0 μW.
- i State one source of attenuation in an optical fibre. [1]
 - ii Determine the distance after which the signal must be amplified. [3]

- HL** 10 a i State what is meant by **ultrasound**. [1]
- ii Ultrasound and X-rays are equally capable of imaging parts of the body. Suggest why ultrasound would be the preferred method of imaging. [1]
- b The impedance of air is $Z_{\text{air}} = 410 \text{ kg m}^{-2} \text{ s}^{-1}$ and that of soft tissue is $Z_{\text{tissue}} = 1.8 \times 10^6 \text{ kg m}^{-2} \text{ s}^{-1}$.
The fraction of the intensity that gets reflected back from the air–tissue boundary is $\frac{I_r}{I_0} = \frac{(Z_{\text{air}} - Z_{\text{tissue}})^2}{(Z_{\text{air}} + Z_{\text{tissue}})^2}$.
- i Calculate this fraction. [1]
 - ii Comment on the answer to i, suggesting a solution to the problem it reveals. [3]
- c A pulse of ultrasound is reflected from the boundary of an organ 6.5 ms after it is emitted. The region between the surface of the skin, where the pulse originates, and the organ is filled with tissue in which the speed of sound is 1500 ms⁻¹. Estimate the distance of the organ boundary from the surface of the skin. [2]

- HL** 11 a State what is meant by **half-value thickness** (HVT). [1]
- b The half-value thickness of soft tissue for X-rays of a given energy is 4.10 mm.
- i After a distance x in soft tissue, the fraction of the incident intensity of X-rays that gets transmitted is 0.650. Determine this distance. [3]
 - ii State and explain the effect, if any, on the answer to i if X-rays with a larger half-value thickness were to be used. [2]
- c Outline how, in X-ray imaging, the following are achieved:
- i reduction of the blurring in the image [2]
 - ii reduction of the exposure time. [2]

- HL** 12 Outline the technique of **magnetic resonance imaging**. [6]